2025 Level 3, Volume 2, Portfolio Construction PDF (pp. 265 – 268)

Delete entire section entitled "The Impact of Different Tax Rates, Sources of Return, and Inflation"

Replace with the following:

The Impact of Different Sources of Returns with Taxes

The prior examples demonstrate the basic mechanics of tax calculations in a simplified manner, with a single constant return exposed to a single, uniform, unchanging tax rate. In reality, portfolio returns often originate from various sources and may be subject to different tax rates. These rate of return and tax rate issues have implications for portfolio management.

A more comprehensive return model incorporates annual income component, R_{INC}, taxed yearly at the tax rate, t_X, and capital appreciation, $R_{CAPITAL}$, taxed upon liquidation at the capital gains rate, t_{CG}. Incorporating the after-tax effects on annual returns and the capital gains tax upon investment liquidation on the capital gains portion of an investment's return, we arrive at Equation 7 below as a future value accumulation factor:

$$\text{FVIF}_{\text{INC,CAPITAL,TX,TCG}} = \left[1 + \left(R_{INC} \times (1 - t_X)\right) + R_{CAPITAL}\right]^T - \left[\frac{R_{CAPITAL}}{R_{INC} \times (1 - t_X) + R_{CAPITAL}} \times \left[\left[1 + \left(R_{INC} \times (1 - t_X)\right) + R_{CAPITAL}\right]^T - 1\right] \times t_{CG}\right] \quad (7)$$

Equation 7 may be explained as consisting of two primary elements as described below:

- The first bracketed term is the future value of the investment after incorporating the annual tax on income gains.
- The second term in brackets consists of three intermediate terms.
    - The first intermediate term is the ratio of the capital gain return to the total after-tax annual return on income and capital gains.
    - The second intermediate term is a computation of the total gain on the investment.

- The third intermediate term is the tax rate on capital gains.

  Multiplying the three intermediate terms comprising the second primary element provides the total capital gains tax which is deducted from the investment's future value after incorporating annual taxes on income gains.

An application of Equation 7 is demonstrated in the following case study.

BEGIN CASE STUDY

Nataliia Kozlowska: Comparing After-Tax Returns on Different Return Sources and Tax Rates

Continuing with the facts in this series of case studies, Ms. Kozlowska's EUR100,000 investment in the stock expects to generate 5% in nominal annual price appreciation and 2% in nominal annual dividend income. The dividend income is taxed at 20% annually, while the price appreciation is also taxed at 20% upon investment liquidation. Ms. Kozlowska has an investment time horizon of 20 years. For purposes of this example, we assume no inflation.

Questions:

1. Calculate Ms. Kozlowska's expected wealth in after-tax terms after 20 years.

   Answer:

   Ms. Kozlowska's expected wealth is expected to equal EUR319,792.40 after 20 years as described below.

   Equation 7 is shown below and the solution process discusses the different components of the calculation.

$$\text{FVIF}_{\text{INC,CAPITAL,TX,TCG}} = \left[1 + \left(R_{INC} \times (1 - t_X)\right) + R_{CAPITAL}\right]^T - \left[\frac{R_{CAPITAL}}{R_{INC} \times (1 - t_X) + R_{CAPITAL}} \times \left[\left[1 + \left(R_{INC} \times (1 - t_X)\right) + R_{CAPITAL}\right]^T - 1\right] \times t_{CG}\right]$$

As a starting point, we compute that the after-tax annual rate of return implied by RINC and RCAPITAL is 6.6% per year.

$$Annual\ after\text{-}tax\ rate\ of\ return = 2\% \times (1 - 20\%) + 5\% = 6.6\%$$

Thus, the future value accumulation factor for an investment generating 6.6% per year for 20 years is 3.5904104. This is the first bracketed term in the equation above.

$$[1 + (2\% \times (1 - 20\%) + 5\%]^{20} = 3.5904104$$

Next, we compute the amount of the capital gains tax by employing the three components of the second primary element of the equation.

- Ratio of the capital gain return to the total after-tax annual return on income and capital gains = 5% ÷ 6.6% = 75.76%
- Total gain on the investment = 2.5904104
- Capital gains tax rate = 20%

$$Total\ capital\ gains\ tax\ =\ 0.7576\ x\ 2.59041\ x\ 0.20\ =\ 0.3924864$$

$$\text{FVIF}_{\text{INC,CAPITAL,TX,TCG}} = 3.5904104 - 0.3924864 = 3.197924$$

Multiplying the above result by the initial investment of EUR100,000 provides Ms. Kozlowska's expected wealth in after-tax terms after 20 years of EUR319,792.40.

2. Suppose that Ms. Kozlowska has a different investment opportunity which expects to generate 2% in nominal annual price appreciation and 5% in nominal annual dividend income. Calculate Ms. Kozlowska's expected wealth in after-tax terms after 20 years.

   Answer:

   Following the same process as illustrated in the calculations to the prior scenario, we find that Ms. Kozlowska's expected after-tax wealth for this investment is EUR306,001 as shown below.

   In this case, the after-tax annual rate of return implied by RINC and RCAPITAL is 6.0% per year.

$$Annual\ after\text{-}tax\ rate\ of\ return = 5\% \times (1 - 20\%) + 2\% = 6.0\%$$

The future value accumulation factor for an investment generating 6.0% per year for 20 years is 3.20713547.

$$[1 + (5\% \times (1 - 20\%) + 2\%]^{20} = 3.20713547$$

The three components of the second primary element of the equation are as follows:

- Ratio of the capital gain return to the total after-tax annual return on income and capital gains = 2% ÷ 6% = 33.33%
- Total gain on the investment = 2.20713547
- Capital gains tax rate = 20%

$$Total\ capital\ gains\ tax\ =\ 0.3333\ x\ 2.20713547\ x\ 0.20\ =\ 0.14712765$$

$$\text{FVIF}_{\text{INC,CAPITAL,TX,TCG}} = 3.20713547 - 0.14712765 = 3.06001$$

Multiplying the above result by the initial investment of EUR100,000 provides Ms. Kozlowska's expected wealth in after-tax terms after 20 years of EUR306,001.

3. Contrast the after-tax wealth implications of the two investments above by discussing the tax drag associated with the two investments considered in the two prior questions.

Answer:

To assess the tax drag of these two investments, we first note that, in the absence of taxes, both investments generate 7% in annual return. Without taxes, the after-tax wealth of EUR100,000 invested for 20 years at 7% is equal to EUR386,968.44 as shown below.

$$100{,}000\ \times [1 + 0.07]^{20} = 386{,}968.44$$

The tax drag is reflected by the difference between the after-tax wealth compared to the wealth in the absence of taxes as a percentage of the total gain in the absence of taxes. Ms. Kozlowska is clearly better off from an after-tax perspective with the investment opportunity implied by the first question in which income taxed annually generates the

lower return of 2% per year while the higher capital gains return of 5% is not taxed until the end of the 20-year investment horizon.

$$Tax\ drag\ of\ first\ scenario\ = \frac{386{,}968.44 - 319{,}792.40}{286{,}968.44} = 23.4\%$$

$$Tax\ drag\ of\ second\ scenario\ = \frac{386{,}968.44 - 306{,}001}{286{,}968.44} = 28.2\%$$

Effectively, the second scenario creates a larger total tax burden because more of each year's return is taxed. The first scenario featured 6.6% annual return after-tax while the second scenario only generated 6.0% annual return after-tax. Despite lower capital gains taxes in the second scenario, the compounding power of the additional 60 basis points of after-tax annual return sufficiently offsets the additional capital gains taxes paid in the first scenario.

END CASE STUDY