READING

5

Return Attribution

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LEARNING OUTCOMES

Mastery	*The candidate should be able to:*
☐	**a.** explain purposes of return attribution and the role of return attribution in the investment decision-making process;
☐	**b.** distinguish between return attribution and return contribution analysis;
☐	**c.** distinguish between return attribution and risk attribution;
☐	**d.** describe the attributes of an effective attribution process;
☐	**e.** analyze the sources of performance of a portfolio using the Brinson–Hood–Beebower and Brinson–Fachler models;
☐	**f.** calculate and interpret arithmetic allocation, selection, and interaction attribution effects;
☐	**g.** explain the use of an interaction effect, including its advantages and disadvantages;
☐	**h.** calculate and interpret geometric allocation, selection, and interaction attribution effects;
☐	**i.** describe returns-based, holdings-based, and transactions-based attribution, including the advantages and disadvantages of each;
☐	**j.** distinguish between the effects of sponsors' and managers' investment decisions;
☐	**k.** calculate and interpret attribution analysis at different levels: plan sponsor, portfolio manager, country, industrial sector, and individual security;
☐	**l.** interpret the results of a factor model–based return attribution analysis;
☐	**m.** compare Brinson models (asset-grouping models) with factor models of attribution, including the advantages and disadvantages of each;
☐	**n.** explain why the standard Brinson approach may not be suitable for fixed-income strategies;
☐	**o.** describe the different types of fixed-income attribution models and interpret the results of a fixed-income attribution analysis;

(continued)

LEARNING OUTCOMES

Mastery	The candidate should be able to:
☐	**p.** explain the inputs necessary for a holdings-based and a transactions-based return attribution analysis and the problems associated with each;
☐	**q.** explain possible causes of residuals in attribution analysis;
☐	**r.** calculate and explain off-benchmark (zero-weight sector) attribution effects.

1 INTRODUCTION

A previous reading explained how rate of return is defined, measured, and interpreted. Investment performance measurement—establishing the returns earned by a portfolio—is essentially a descriptive process. Performance analysts also need to analyze performance records to evaluate how the portfolio was managed and which decisions were profitable or unprofitable. Questions they may want to answer about a portfolio include the following:

- Did the portfolio's return exceed, equal, or fall short of that of its assigned benchmark (reference point)?
- What active sector (e.g., industry) weighting decisions did the portfolio manager make? That, is, which sectors did he or she overweight (underweight) relative to the benchmark's weightings?
- Were active sector decisions profitable?
- Did the manager seek profits through individual security selection?
- Were individual security selection decisions profitable?
- Was the way the portfolio manager sought to add value consistent with the manager's self-description of his/her investment discipline?

The techniques of return attribution can provide answers to such questions and any question that addresses the consequences of investment decisions. Return attribution analysis is particularly important when performance is weak; portfolio managers must demonstrate an understanding of their performance, provide a rationale for their decisions, and generate confidence in their ability to add value in the future. Return attribution provides quality control for the investment process across asset management firms, illuminating key strengths and weaknesses essential to managing a complex business with multiple investment strategies. Return attribution also provides information that is helpful for investigating investment management skill—the subject of performance appraisal.

The balance of this reading is organized as follows. Section 2 provides an overview of the various types of return attribution analysis and their purposes. Section 3 presents arithmetic return attribution models, including the foundation Brinson models, and Section 4 presents geometric return attribution models. Section 5 contrasts holdings-based and transactions-based return attribution. Section 6 discusses the variations in the number of levels at which return attribution is performed. Section 7 introduces factor-based return attribution. The return attribution modeling presented up to Section 7 is developed with an equity focus; Section 8 offers a concise introduction to fixed-income return attribution, often considered to be a specialist area. Section 9 addresses several common problems in return attribution. Section 10 concludes and provides a summary.

2 RETURN ATTRIBUTION: AN OVERVIEW

To evaluate the performance of an actively managed portfolio, analysts often compare the portfolio's returns with those of its assigned benchmark. If we assume that the benchmark return represents the performance available from a passive investment in some appropriately selected segment of the market, then the difference between the performance return and the benchmark return represents the performance as a result of active investment decisions. The difference between the return on a portfolio and the return on its assigned benchmark is known in the investment performance field as the portfolio's **excess return**.[1] In this reading, outperformance (underperformance) is sometimes used to refer to positive (negative) excess return with respect to a benchmark. A dictionary definition of the verb "attribute" is "to explain something by indicating a cause." Return attribution can be defined as follows:

- **Return attribution** is a set of techniques used to identify the sources of the excess return of a portfolio against its benchmark in order to understand the consequences of active investment decisions.

2.1 Purposes of Return Attribution

Return attribution is part of the feedback loop of the portfolio management process, quantifying active decisions of portfolio managers, monitoring consistency, and informing senior management and clients. As a feedback mechanism, return attribution can be thought of as "backward looking" or *ex post*, meaning that it is used to evaluate the investment decisions for some historical time horizon. Return attribution allows us to look across a specific time horizon and identify which investment decisions have either added or detracted value from the portfolio, relative to its benchmark.

A Common Type of Return Attribution

Suppose a portfolio's return for the past year was 5.24% and the portfolio's benchmark return for that same time period was 3.24%. In this case, the portfolio achieved a positive arithmetic excess return of 2.00% (5.24% – 3.24% = 2.00%) over the past year.

Return attribution can then be applied to understand how the 2.00% was achieved. Was the return achieved by selecting securities that performed well relative to the benchmark or avoiding benchmark securities that performed relatively poorly (*security selection*)? Or was the return achieved by choosing to over-invest in (or overweight) a particular economic sector or asset category that outperformed the total benchmark for that period or to underinvest in or avoid (or underweight) an asset category that underperformed the total benchmark (*asset allocation*)?

1 The term *active return* is also frequently used to describe the excess return of a portfolio in relation to the benchmark return. We are consistent in using *excess return* only in the one sense given in the current reading; in other contexts, the reader should be aware that it can refer to a return in excess of the risk-free return.

Models of equity return attribution often attempt to separate the investment process into those two key decisions—selection and allocation—assigning both a magnitude and direction (plus or minus) for both decisions. For instance, using the above data, we might calculate the return attribution results shown in Exhibit 1:

Exhibit 1 Total Portfolio Return Attribution Analysis (Time Period: Past 12 Months)

Portfolio Return	Benchmark Return	Excess Return	Allocation Effect	Selection Effect
5.24%	3.24%	2.00%	–0.50%	2.50%

As we noted, the investment decisions generated a positive excess return of 200 basis points (bps) relative to the benchmark. We use the return attribution analysis to see how this 200 bps was generated. First, we should note that the *negative* allocation effect indicates that the asset allocation decisions over the past 12 months, whatever they were, had a negative impact on the total portfolio performance. They *subtracted* 50 bps from the excess return. In contrast, the *positive* selection effect indicates that the security selection decisions—decisions to overweight or underweight securities relative to their benchmark weights—*added* 250 bps to the excess return. Our return attribution analysis implies that the portfolio manager's security selection decision was far superior to his or her asset allocation decision for the time period examined.

When conducting a return attribution analysis, we need to also consider the different perspectives important to different roles within the investment process. The fund sponsor, for example, will be interested in different parts of the investment process than the portfolio manager because the fund sponsor will delegate some portion of the management to the portfolio manager. The fund sponsor will want to understand if any added value from the portfolio manager is consistent through time and consistent with the manager's stated investment discipline. Any added value derived from a source other than that explicitly described in the investment process may be random or not repeatable. Return attribution analysis can provide evidence in support of the claimed competencies of the portfolio manager. Prospective clients will want evidence of the investment process supported by return attribution analysis.

Return attribution may also identify other problems, such as holding too much cash in a rising market or unnecessarily high transaction costs when implementing part of the decision process.

2.2 Return Attribution vs. Return Contribution Analysis

Occasionally, performance analysts may conduct an *absolute* return attribution analysis, also known as return contribution analysis. We refer to it as absolute return attribution because, unlike return attribution as ordinarily understood, it is not calculated relative to a benchmark. Return contribution analysis uses only the weights and returns of the portfolio (without reference to the weights and returns of a benchmark).

- **Return contribution analysis** (**absolute return attribution**) identifies the contributions of portfolio components to the total return of a portfolio.

For return contribution analysis, we look at portfolio weights, component security or sector returns, and the weighted return or *contribution to return*. Contribution to return is calculated as the product of the security or sector weight multiplied by the security/sector return, as in:

$$R = \sum_{i=1}^{i=n} w_i R_i$$

where

n = the number of sectors or securities in the portfolio
w_i = the weight of the sector or security in the portfolio
R_i = the return of the sector or security in the portfolio
$w_i R_i$ = the contribution to portfolio return

The sum of the contributions to return is equal to the total portfolio return, R. Consider the example of a portfolio containing the three securities listed in Exhibit 2.

Exhibit 2 Return Contribution Analysis

	Weight (%)	Return (%)	Contribution(%)
Security A	25	4.80	1.20
Security B	50	2.50	1.25
Security C	25	−1.20	−0.30
Portfolio Total	100	2.15	Sum = 2.15

For each security, there is a weight, a return, and a contribution to return. The sum the contributions to return gives the total portfolio return: 1.20 + 1.25 + −0.30 = 2.15.

This return contribution analysis indicates that securities A and B made similar contributions to the total return (1.20 and 1.25 respectively). Although security B had a much larger weight in the portfolio (50%) than security A (25%), security B's significantly smaller return (2.5% versus 4.8%) produced a contribution almost equal to security A. Security C, with a negative return, had a negative contribution to the total portfolio return.

Return contribution analysis can tell us which securities have the greatest (and least) impact on the total portfolio return. But, as noted, the analysis does not include a comparison to a performance benchmark. So, although contribution analysis provides some insight into the *absolute* impact of individual securities (or groups of securities) on the portfolio, it is not informative about whether investment decisions added value *relative* to the benchmark.

EXAMPLE 1

Return Contribution Analysis

Return contribution analysis can be used to:

A measure the investment risk relative to the benchmark.

B compare the relative impact of securities within a portfolio.

C identify the investment value added from the asset weighting decisions relative to the benchmark.

Solution:

B is correct. Return contribution analysis is used to compare the weighted returns of separate investments within a portfolio, thus allowing the impact of those separate investments to be compared.

2.3 Return Attribution vs. Risk Attribution

Complementary to return attribution is risk attribution. Whereas return attribution analyzes the consequences of active investment decisions on returns, risk attribution analyzes the risk consequences of such decisions. Depending on the purpose of the analysis, risk may be viewed in absolute or benchmark-relative terms. For example, when risk relative to a benchmark is the focus, a risk attribution analysis might identify and evaluate a portfolio's deviations from a benchmark's exposures to risk factors. Risk attribution is presented in detail in a separate reading.

Performance attribution is defined to include return attribution and risk attribution (although in practice "performance attribution" is frequently used to just mean "return attribution").

EXAMPLE 2

Return and Risk Attribution

1 Return attribution can *best* be used to:

- A measure volatility within a portfolio.
- B adjust performance returns for external cash flows.
- C analyze the value added by active investment decisions.

2 Return attribution attempts to identify investment management value added by:

- A identifying which security selection decision was the best overall within the portfolio.
- B focusing on the analysis of holdings that have made the greatest contribution to return.
- C decomposing the excess return into the separate contributors to excess return from allocation and selection decisions relative to the benchmark.

3 Risk attribution is *best described* as concerned with identifying:

- A the level of risk in a portfolio.
- B contributions to a portfolio's alpha risk.
- C the contributors to risk either in a benchmark-relative or absolute sense.

Solution to 1:

C is correct. Return attribution attempts to analyze whether value was added by active investment decisions.

Solution to 2:

C is correct. A typical return attribution analysis includes a decomposition of the excess return into the excess return generated by the asset allocation and security selection decisions separately.

Solution to 3:

C is correct. Risk attribution provides insight into the key contributors to risk.

2.4 Effective Return Attribution

Although first developed as an aid for portfolio management, return attribution analysis is equally useful for senior management, client relationship specialists, risk controllers, operations staff, and sales and marketing professionals on the one hand and clients and prospective clients on the other hand. In identifying the sources of excess return, it is the tool that allows performance analysts to add value and to participate in the investment decision process.

In effect, return attribution analysis is the tool that converts performance measurement information from the back office to information that is useful to the middle office control function. Effective return attribution analysis requires a deep understanding of the investment decision process; return attribution must reflect the active decisions of the portfolio manager. There is little value in analyzing factors that are not part of the investment decision process.

An effective return attribution process must:

- reconcile to the total portfolio return,
- reflect the investment decision process,
- quantify the active decisions of the portfolio manager, and
- provide a complete understanding of the excess return of the portfolio.

If the return generated by the return attribution analysis does not reconcile to the return presented to the client, then at best the return attribution is incomplete and at worst the quality of the return attribution analysis is brought into doubt. If the return attribution does not reflect the investment decision process, then the analysis will be of little value to either the portfolio manager or client. For example, if the portfolio manager is a genuine bottom up stock picker who ignores sector benchmark weights, then any value in measuring the impact of sector allocation against these weights relates not to measuring success in stock picking but to gauging the unintentional sector return effects of the manager's investment discipline.

Return attribution provides a good starting point for a dialogue with clients, explaining both positive and negative aspects of recent performance. In fact, return attribution analysis is particularly important when performance is weak; portfolios managers must demonstrate an understanding of their performance, provide a rationale for their decisions, and generate confidence in their ability to add value in the future. When it accurately reflects the investment decision-making process, return attribution provides quality control for the investment process; it provides senior management with the tool they need to manage a complex business with multiple investment strategies.

EXAMPLE 3

Effective Return Attribution

An effective return attribution process is best described as:

A adjusting fully for risk.

B identifying poor performance.

C quantifying the investment decision process.

Solution:

C is correct. Return attribution analysis does not focus on adjusting for risk, nor does it attempt to identify poor performance alone. Return attribution may be effective if it quantifies and thus reflects the investment decision process.

3 ARITHMETIC EQUITY RETURN ATTRIBUTION

The foundations of return attribution were established in two articles written by Brinson and Fachler (1985) and Brinson, Hood, and Beebower (1986); today these are collectively known as defining the **Brinson model**. The Brinson model could be called an **asset-grouping model** in the sense that it isolates attributions effects by comparing the returns of variously constructed portfolios or groups of assets.

These articles build on the assumption that the total portfolio and benchmark returns can be disaggregated as follows:

$$\text{Portfolio return } R = \sum_{i=1}^{i=n} w_i R_i \tag{1}$$

$$\text{Benchmark return } B = \sum_{i=1}^{i=n} W_i B_i \tag{2}$$

where

w_i = weight of the ith sector in the portfolio
R_i = return of the portfolio assets in the ith sector
W_i = weight of the benchmark in the ith sector
B_i = return of the benchmark in the ith sector
n = number of sectors or securities[2]

The sum of the weights in both the portfolio and benchmark is required to equal 100%. That is,

$$\sum_{i=1}^{i=n} w_i = 1 \text{ and } \sum_{i=1}^{i=n} W_i = 1$$

If the weights of the portfolio sum to less than 100%, it means some part of the portfolio is missing and the analysis will be incomplete. If the weights of the portfolio sum to greater than 100%, it means that either the total value of the portfolio is incorrect or the value of one or more sectors is incorrect. The total sum of the parts of the portfolio cannot be greater than the total portfolio size. The presence of leverage would require a negative weight (borrowings or short positions) to balance to 100%.

At this stage, we are concerned with only single period, single currency return attribution models. Multi-period, multi-currency models will be covered in other readings.

The challenge for attribution analysis is to quantify each of the portfolio manager's active decisions that explain the difference between the portfolio return R and the benchmark return B.

Exhibit 3 provides data for a three-sector domestic equity portfolio.

2 n is taken to refer to a number of sectors or securities depending on context. Null (zero weight) holdings are counted in n. For sectors, the number should be the same for portfolios and their benchmarks. The count of securities, however, will generally vary between a portfolio and its benchmark. In summations involving a portfolio and its benchmark, n can be understood to be the number of unique holdings that are in either the portfolio or the benchmark, counting cash as a holding.

Exhibit 3 Brinson Model Illustration

Sector	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Energy	50%	50%	18%	10%
Health Care	30%	20%	−3%	−2%
Financials	20%	30%	10%	12%
Total	**100%**	**100%**	**10.1%**	**8.2%**

The total portfolio return $R = \sum_{i=1}^{i=n} w_i R_i$ is:

$$R = 50\% \times 18\% + 30\% \times -3\% + 20\% \times 10\% = 10.1\%$$

The total benchmark return $B = \sum_{i=1}^{i=n} W_i B_i$ is:

$$B = 50\% \times 10\% + 20\% \times -2\% + 30\% \times 12\% = 8.2\%$$

Thus, 10.1% − 8.2% = 1.9%. A return difference of 190 bps between the portfolio's and the benchmark's returns needs to be attributed to the portfolio manager's decision.

3.1 The Brinson–Hood–Beebower (BHB) Model

Brinson, Hood, and Beebower (1986) presented a breakdown of the arithmetic excess return assuming a simple two-step investment decision process in which the portfolio manager seeks to add value through both allocation and selection.

In return attribution, **allocation** refers to the value the portfolio manager adds by having different sector weights in the portfolio than the sector weights in the benchmark. A sector weight in the portfolio greater than the equivalent benchmark sector weight would be described as *overweight* and a lesser weight would be described as *underweight.*

Clearly, the portfolio manager will aim to overweight outperforming sectors and underweight underperforming sectors. In their original article, Brinson, Hood, and Beebower called this effect *timing. Allocation* is a more appropriate, and now more common, label.

Selection refers to the value the portfolio manager adds by holding individual securities or instruments within the sector in different-than-benchmark weights. The portfolio manager making selection decisions may or may not be the same portfolio manager making the allocation decisions. In fact, allocation decisions may often be made collectively by an asset allocation committee.

Again, the portfolio manager will aim to overweight outperforming securities relative to their respective benchmark and underweight underperforming securities relative to their respective benchmark.

3.1.1 *Allocation*

To identify the added value from allocation, we will calculate the return of an intermediate fund called the "allocation notional fund," which is one step away from the benchmark portfolio and one step toward the actual portfolio. The term notional is used in this context to describe the hypothetical return that results from the portfolio manager's allocation decisions but includes none of the selection decisions.

In the allocation notional fund, the sector weights of the actual fund are applied to the benchmark returns within each sector. By definition, the return on this notional fund includes all the portfolio manager's sector allocation decisions but excludes all individual security selection decisions (because benchmark returns are used within the sector).

Allocation notional fund return[3] is

$$B_S = \sum_{i=1}^{i=n} w_i B_i \tag{3}$$

The allocation notional return for the data from Exhibit 3 is:

$B_S = 50\% \times 10\% + 30\% \times -2\% + 20\% \times 12\% = 6.8\%$

The contribution from asset allocation is the difference between the allocation notional fund return and the benchmark return or:

$$B_S - B = \sum_{i=1}^{i=n} w_i B_i - \sum_{i=1}^{i=n} W_i B_i = \sum_{i=1}^{i=n} (w_i - W_i) B_i \tag{4}$$

The contribution to allocation in the ith sector is

$$A_i = (w_i - W_i) B_i \tag{5}$$

Note that the sum of sector contributions to allocation equals the arithmetic excess return from allocation:

$$\sum_{i=1}^{i=n} A_i = B_S - B \tag{6}$$

The contribution to arithmetic excess return from allocation for the portfolio data shown in Exhibit 3 is $B_S - B = 6.8\% - 8.2\% = -1.4\%$. Individual sector allocation effects $A_i = (w_i - W_i)B_i$ are as follows:

Energy	(50% – 50%) × 10% = 0.0%
Health care	(30% – 20%) × –2.0% = –0.2%
Financials	(20% – 30%) × 12% = –1.2%
Total	0.0% – 0.2% – 1.2% = –1.4%

The portfolio weight in the Energy sector is in line with the benchmark weight, therefore, there is no contribution to allocation in this sector.

There is, however, an overweight position of 10% in the Health Care sector, which when applied to the negative benchmark return of –2.0%, results in a contribution of –0.2%. The manager has overweighted a poorly performing sector.

It follows, if there is an overweight sector, there must be a least one other underweight sector. The 10% underweight position in Financials combined with a 12.0% benchmark return results in a contribution of –1.2%. The manager has underweighted a strongly performing sector. The total contribution to arithmetic excess return from sector allocation is –1.4%

3 The allocation notional fund will be renamed later to the semi-notional fund, which explains the subscript S in B_S.

EXAMPLE 4

Allocation Using the BHB Model

Exhibit 4 Three-Sector Portfolio Example

Sector	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Technology	20%	30%	–11.0%	–10.0%
Telecommunications	30%	40%	–5.0%	–8.0%
Utilities	50%	30%	–8.0%	–5.0%
Total	**100%**	**100%**	**–7.7%**	**–7.7%**

Using the BHB model, the allocation effect of Utilities based on the portfolio data in Exhibit 4 is:

A –1.6%

B –1.5%

C –1.0%

Solution:

C is correct. $(w_i - W_i)B_i$ = (50% – 30%)(–5.0%) = –1.0%. The portfolio was 20% overweight in a sector delivering –5% performance, thus contributing –1% to the overall allocation effect.

A is incorrect because $(w_i - W_i)R_i$ = (50% – 30%)(–8.0%) = –1.6%. The portfolio return of –8% has been used rather than the benchmark return.

B is incorrect because W_iB = 30%(–5.0%) = –1.5% represents only the contribution to the benchmark return from Utilities.

3.1.2 *Selection*

To identify the added value from selection, we will calculate the return of a different intermediate fund called the "selection notional fund," which is also one step away from the benchmark return but isolates a different decision of the portfolio manager in the investment decision process. In the selection notional fund, the sector weights of the benchmark are maintained and applied to the sector returns achieved by the portfolio manager. By definition, the return on this notional fund includes the portfolio manager's individual selection decisions, but excludes any contribution from allocation effects.

Selection notional fund return is

$$R_S = \sum_{i=1}^{i=n} W_i R_i \quad (7)$$

Again using the data from Exhibit 3, the selection notional return is:

R_S = 50% × 18% + 20% × –3% + 30% × 10% = 11.4%

The contribution from selection is the difference between the selection notional fund return and the benchmark return:

$$R_S - B = \sum_{i=1}^{i=n} W_i R_i - \sum_{i=1}^{i=n} W_i B_i = \sum_{i=1}^{i=n} W_i \times (R_i - B_i) \quad (8)$$

The contribution to selection in sector i is:

$$S_i = W_i\left(R_i - B_i\right) \quad (9)$$

Note that:

$$\sum_{i=1}^{i=n} S_i = R_S - B \quad (10)$$

Exhibit 3 Brinson Model (BHB) Illustration (repeated)

Sector	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Energy	50%	50%	18%	10%
Health Care	30%	20%	–3%	–2%
Financials	20%	30%	10%	12%
Total	**100%**	**100%**	**10.1%**	**8.2%**

For the portfolio data shown in Exhibit 3, the contribution to arithmetic excess return from selection is $R_S - B$ = 11.4% – 8.2% = 3.2%. Sector selection effects $S_i = W_i(R_i - B_i)$ are as follows:

Energy	50% × (18% – 10%) = 4.0%
Health Care	20% × (–3.0% + 2.0%) = –0.2%
Financials	30% × (10.0% – 12.0%) = –0.6%
Total	4.0% – 0.2% – 0.6% = 3.2%

Selection within the Energy sector added 800 bps to the benchmark sector return; applying the benchmark weight of 50% to this sector resulted in a 4.0% contribution to arithmetic excess return.

Selection in the Health Care sector subtracted 100 bps from the sector benchmark return sector; applying a 20% weighting thus resulted in a contribution of –0.2%.

Selection within Financials was also poor, subtracting 200 bps from the benchmark sector return; applying a 30% weighting thus generated a contribution of –0.6%.

Total contribution to arithmetic excess return from selection is 3.2 percentage points.

Combining allocation of –1.4% and selection of 3.2%, 1.8% of total added value of 1.9% is explained and 0.1% is so far unexplained. This 0.1% contribution to excess return is explained by the interaction of selection and allocation decisions.

EXAMPLE 5

Selection Using the BHB Model

Exhibit 5 The BHB Model and Selection

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Materials	30%	20%	12.0%	8.0%
Industrials	30%	20%	8.0%	10.0%

Exhibit 5 (Continued)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Consumer Staples	40%	60%	5.0%	8.0%
Total	**100%**	**100%**	**8.0%**	**8.4%**

Using the BHB model, the selection effect of Industrials based on the portfolio data shown in Exhibit 5 is:

A –2.0%

B –0.6%

C –0.4%

Solution:

C is correct. $W_i(R_i - B_i)$ = (20%)(8.0% – 10.0%) = –0.4%. In the industrials sector the portfolio manager underperformed by 2%. Applying the benchmark weight of 20%, the contribution to selection from Industrials is –0.4%.

A is incorrect because –2.0% is simply the arithmetic difference of return in the industrials sector: 8.0% – 10.0%= –2.0%.

B is incorrect because $w_i(R_i - B_i)$ = (30%)(8.0% – 10.0%) = –0.6%. The portfolio weight has been used rather than the benchmark weight.

3.1.3 *Interaction*

In the BHB model, selection and allocation do not explain the arithmetic difference completely. For example, in the attribution analysis based on Exhibit 3, allocation (–1.4%) and selection (3.2%) together represent just 1.8 percentage points of the arithmetic difference between the portfolio return of 10.1% and the benchmark return of 8.2%; 0.1% (i.e., 10 bps) is missing. Thus, a third term, which will be called interaction, is required:

$$\text{Selection} + \text{Allocation} = (R_S - B) + (B_S - B)$$

or

$$= R_S + B_S - 2B \neq R - B$$

To achieve attribution factors that add up to the arithmetic difference between the portfolio and benchmark returns, we must introduce a third term called interaction:

$$\underbrace{R_S - B}_{\text{Selection}} + \underbrace{B_S - B}_{\text{Allocation}} + \underbrace{R - R_S - B_S + B}_{\text{Interaction}} = R - B \quad (11)$$

In their article, Brinson, Hood, and Beebower described this term as *Other;* interaction is perhaps a better description and is in common usage today. Interaction is not a residual, rather it is a directly calculable effect resulting from the combination of (or interaction between) allocation and selection effects, so that:

$$R - R_S - B_S + B = \sum_{i=1}^{i=n} w_i R_i - \sum_{i=1}^{i=n} W_i R_i - \sum_{i=1}^{i=n} w_i B_i + \sum_{i=1}^{i=n} W_i B_i$$

The right hand side simplifies to

$$\sum_{i=1}^{i=n} (w_i - W_i)(R_i - B_i) = \sum_{i=1}^{i=n} I_i \quad (12)$$

defining the contribution of sector i to interaction I_i as $I_i = (w_i - W_i)(R_i - B_i)$ or (Sector weighting difference)$_i$ × (Sector return difference)$_i$. Thus,

$$\sum_{i=1}^{i=n} I_i = R - R_S - B_S + B \quad (13)$$

Again using the portfolio data from Exhibit 3:

$R - R_S - B_S + B$ = 10.1% − 11.4% − 6.8% + 8.2% = 0.1%

Individual sector interaction effects, calculated as $I_i = (w_i - W_i)(R_i - B_i)$, are

Energy	(50% − 50) × (18% − 10%) = 0.0%
Health Care	(30% − 20%) × (−3.0% + 2.0%) = −0.1%
Financials	(20% − 30%) × (10.0% − 12.0%) = 0.2%
Total	0.0% − 0.1% + 0.2% = 0.1%

The overall contribution from interaction is small in this case. But the interaction effect need not be small; it is the combination of allocation and selection decisions and typically, although not necessarily, will be greater for larger asset allocation bets.

For the Energy sector, the portfolio weight is in line with the benchmark weight and thus there is no contribution to interaction.

For the Health Care sector, there is an allocation overweight decision of +10%. We have 10% more of portfolio value allocated to this underperforming sector than the benchmark has; because the manager had a negative selection in this sector, a negative contribution from interaction of −0.1% resulted.

In Financials, there is an underweight decision of 10% in this sector in which the manager also showed negative selection. We have less of this sector in which the manager underperforms the benchmark; therefore, the combined effect of being underweight in an underperforming sector is an added value of +0.2%. Total contribution from interaction is +0.1%.

The return attribution results are summarized in Exhibit 6.

Exhibit 6 BHB Return Attribution Results

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection	Interaction
Energy	50%	50%	18%	10%	0.0%	4.0%	0.0%
Health Care	30%	20%	−3%	−2%	−0.2%	−0.2%	−0.1%
Financials	20%	30%	10%	12%	−1.2%	−0.6%	0.2%
Total	**100%**	**100%**	**10.1%**	**8.2%**	**−1.4%**	**3.2%**	**0.1%**

EXAMPLE 6

Interaction

Exhibit 7 Sample Portfolio Data

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Materials	30%	20%	12.0%	8.0%
Industrials	30%	20%	8.0%	10.0%
Consumer Staples	40%	60%	5.0%	8.0%
Total	**100%**	**100%**	**8.0%**	**8.4%**

Using the BHB model, the interaction effect of Industrials based on the portfolio data in Exhibit 7 is:

A –0.20%

B –0.04%

C 0.00%

Solution:

A is correct: $(w_i - W_i)(R_i - B_i)$ = (30% – 20%)(8.0% – 10.0%) = –0.2%. The portfolio is 10% overweight in a sector in which the manager underperformed by 200 bps; hence, there is an additional –0.2% contribution to performance.

B is incorrect: $(w_i - W_i)(R_i - B)$ = (30% – 20%)(8.0% – 8.4%) = –0.4%. Total benchmark return is used instead of benchmark sector return.

C is incorrect: $(w_i - W_i)(R_i - R)$ = (30% – 20%)(8.0% – 8.0%) = 0.00%. Total portfolio return is used instead of benchmark sector return

Exhibit 8 illustrates the Brinson framework for return attribution.

Exhibit 8 Brinson Framework for Return Attribution

		Selection: Actual	Selection: Passive
Allocation	Actual	**Quadrant I** Portfolio Return $R = \sum_{i=1}^{i=n} w_i R_i$	**Quadrant II** Allocation Notional $B_S = \sum_{i=1}^{i=n} w_i B_i$
	Passive	**Quadrant III** Selection Notional $R_S = \sum_{i=1}^{i=n} W_i R_i$	**Quadrant IV** Benchmark Return $B = \sum_{i=1}^{i=n} W_i B_i$

Excess returns are calculated as follows:

Asset allocation II – IV

Security selection III – IV

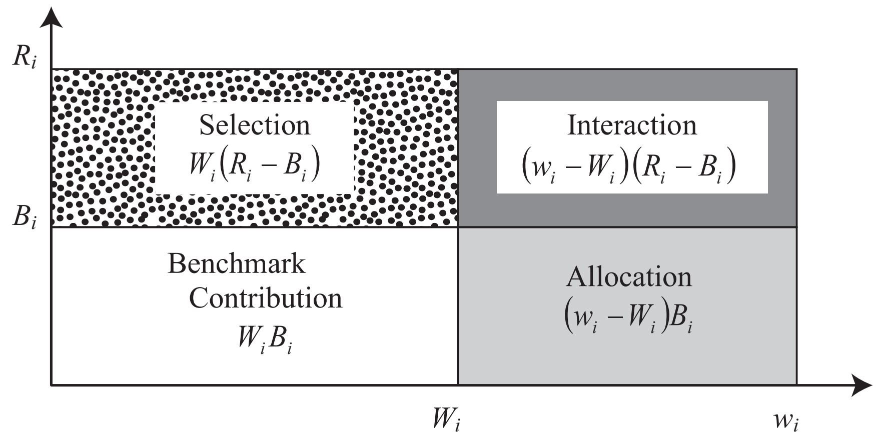
Interaction I – III – II + IV

Total I – IV

Graphically, Exhibit 9 illustrates the attribution factors for each sector i. The contribution to total portfolio return from sector i is the area w_iR_i; the contribution from the benchmark is area W_iB_i. Note that in this particular exhibit, $R_i > B_i$ and $w_i > W_i$.

The contribution to excess return in sector i is the sum of the areas representing Selection $W_i(R_i - B_i)$, Allocation $(w_i - W_i)B_i$, and Interaction $(w_i - W_i)(R_i - B_i)$.

Exhibit 9 Diagram of BHB Model



The BHB model successfully breaks down the sources of arithmetic excess return. But does it reflect the investment decision process of the portfolio manager?

For the most part, asset allocation decisions are made in the context of an overall benchmark return. The portfolio manager is not only seeking to be overweight in markets that make positive returns but also to be overweight in markets that outperform the overall benchmark. The portfolio manager will lose value by being overweight in a market with a positive return if that return is less than the overall benchmark. Therefore, in such a situation, a return attribution model is needed that better reflects the decision process by showing a negative allocation effect.

3.2 Brinson–Fachler Model

The Brinson–Fachler (BF) model differs from the BHB model only in how **individual sector allocation** effects are calculated.

In the BHB model, all overweight positions in sectors with positive returns will generate positive allocation effects irrespective of the overall benchmark return, whereas all overweight positions in negative markets will generate negative allocation effects.[4] Thus, overweighting a sector i that earns a positive return, $B_i > 0$, results in a positive allocation effect, $A_i = (w_i - W_i)\, B_i > 0$, even when the sector return is less than the overall benchmark return (i.e., $B_i < B$). When the sector return is negative, $0 > B_i$, overweighting produces a negative allocation effect, $A_i = (w_i - W_i)\, B_i < 0$.

4 In their original articles, Brinson, Hood, and Beebower and Brinson and Fachler did not emphasize this difference, but over the decades, performance measurement practitioners have attributed slightly different methodologies to each of these papers.

Clearly, if the portfolio manager is overweight in a negative market that has outperformed the overall benchmark, the effect should be positive.

The BF model solves this problem by modifying the asset allocation factor to compare returns with the overall benchmark as follows:

$$B_S - B = \sum_{i=1}^{i=n}(w_i - W_i)B_i = \sum_{i=1}^{i=n}(w_i - W_i)(B_i - B) \quad \textbf{(14)}$$

Because $\sum_{i=1}^{i=n} w_i = \sum_{i=1}^{i=n} W_i = 1$ the constant B can be introduced. The contribution to asset allocation in the ith sector is now:

$$A_i = (w_i - W_i)(B_i - B) \quad \textbf{(15)}$$

Note that in Equation 15 the allocation effect at the portfolio level, $B_S - B$, is unchanged from the BHB model.

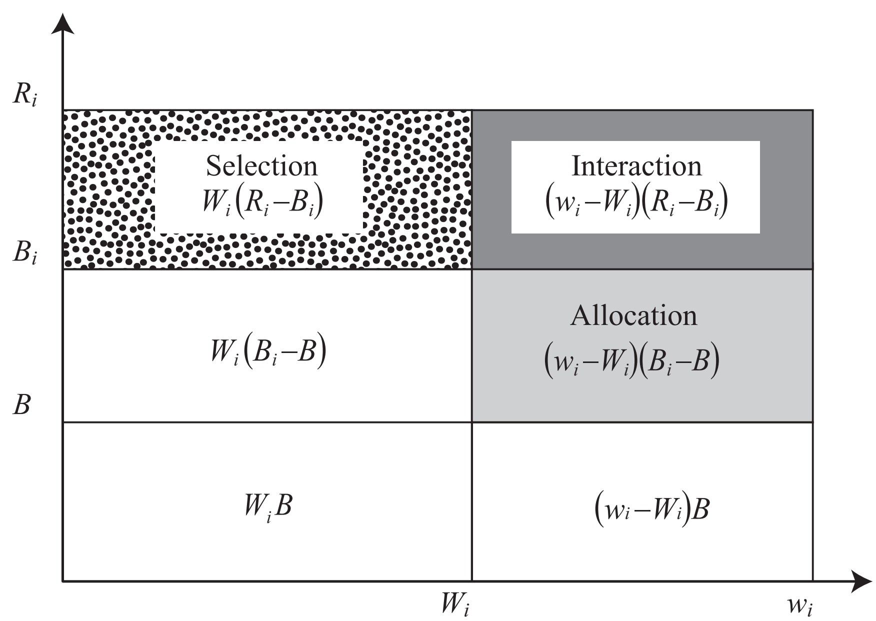
Graphically extending Exhibit 9 to include the benchmark return, in Exhibit 10 we observe no change to the areas representing Selection and Interaction, but Allocation is now described by the area $(w_i - W_i)(B_i - B)$. Note that in this particular exhibit $R_i > B_i > B$ across all sectors because $\sum_{i=1}^{i=n} W_i B_i = B$

then $\sum_{i=1}^{i=n} W_i (B_i - B) = 0$,

and because $\sum_{i=1}^{i=n} w_i = \sum_{i=1}^{i=n} W_i = 1$

then $\sum_{i=1}^{i=n} (w_i - W_i) B = 0$.

Exhibit 10 Diagram of BF Model



The only difference between the two versions of the Brinson model is the calculation of individual sector allocation effects. The BF model is more aligned with most portfolio manager's decision processes and thus far more popular.

The contribution to arithmetic excess return from sector allocation for the portfolio data shown in Exhibit 3 is $B_S - B$ = 6.8% – 8.2% = –1.4%. Revised BF sector allocation effects are calculated for the portfolio data in Exhibit 3 as follows, using $A_i = (w_i - W_i)(B_i - B)$:

Energy	(50% – 50%) × (10% – 8.2%) =0.0%
Health care	(30% – 20%) ×(–2.0% – 8.2%) = –1.02%
Financials	(20% – 30%) × (12% – 8.2%) = –0.38%
Total	0.0% – 1.02% – 0.38% = –1.4%

The impact in Healthcare is much greater. In addition to being overweight in a negative market, which cost –0.2%, the portfolio manager is correctly penalized the opportunity cost of not being invested in the overall market return of 8.2%, generating a further cost of 10% × –8.2% = –0.82% and resulting in a total impact of –1.02%. To describe it another way, the portfolio is 10% overweight in a market that is underperforming the overall market by –10.2% (–2.0% – 8.2%) and generating a loss of –1.02%

The impact in Financials is much smaller. Although being underweight in a positive market cost –1.2%, we must add back the opportunity cost of being invested in the overall market return of 8.2%, generating a contribution of –10% × –8.2% = 0.82% and resulting in a total impact of –0.38%. To describe it another way, the portfolio is 10% underweight in an industry that is outperforming the overall market by 3.8% (12.0% – 8.2%), generating a loss of –0.38%. As expected, at the portfolio level, the allocation effect of –1.4% remains the same as that calculated with the BHB model.

The revised attribution effects, which can be compared with the BHB results in Exhibit 6, are summarized in Exhibit 11. Readers will note that the exhibits differ only in the individual allocation effects.

Exhibit 11 BF Return Attribution Results

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection	Interaction
Energy	50%	50%	18%	10%	0.0%	4.0%	0.0%
Health Care	30%	20%	–3%	–2%	–1.02%	–0.2%	–0.1%
Financials	20%	30%	10%	12%	–0.38%	–0.6%	0.2%
Total	**100%**	**100%**	**10.1%**	**8.2%**	**–1.4%**	**3.2%**	**0.1%**

EXAMPLE 7

Allocation Using the BF Model

Exhibit 12 Sample Portfolio Data

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Technology	20%	30%	–11.0%	–10.0%
Telecommunications	30%	40%	–5.0%	–8.0%

Exhibit 12 (Continued)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Utilities	50%	30%	−8.0%	−5.0%
Total	**100%**	**100%**	**−7.7%**	**−7.7%**

Using the BF method, the allocation effect of Utilities based on the portfolio data in Exhibit 12 is:

A −1.50%

B 0.54%

C 1.35%

Solution:

B is correct: $(w_i - W_i)(B_i - B)$ = (50% − 30%)(−5.0% + 7.7%) = 0.54%. The portfolio was 20% overweight in a sector outperforming the overall benchmark by 2.7%, therefore contributing 0.54% to the overall allocation effect.

A is incorrect: W_iB_i = 30% × −5.0% = −1.5% is the contribution to the benchmark return from Utilities.

C is incorrect: $w_i(B_i - B)$ = 50% × (−5.0% + 7.7%) = +1.35%. Only the portfolio weight of 50% has been used, not the overweight position of 20%.

3.3 Interaction Effect

A shortcoming of both Brinson models is the interaction or "other" term. Interaction is not part of the investment decision process because portfolio managers simply do not seek to add value through interaction decisions. It is a mathematical consequence of the other decisions; the interaction between allocation and selection effects.

For most investment decision processes, the allocation decision comes first and selection decisions follow after the cash has been allocated to the sector.

For genuine bottom-up stock pickers, sector allocation decisions are not made; therefore, the return attribution model should reflect this process and measure the contribution of each stock decision to the overall performance, ignoring allocation effects.

Interaction is not well understood because it is not intuitively part of the investment decision process and is thus often not attributed or interpreted correctly. In presentations of attribution data, it should not be ignored or not shown, randomly allocated to other factors, split proportionally, or simply split 50:50 between selection and allocation. These misuses of interaction would lead to potentially misleading presentations of return attribution effects.

Assuming allocation decisions are made first, the contribution from selection must be:

$$R - B_S = \sum_{i=1}^{i=n} w_i R_i - \sum_{i=1}^{i=n} w_i B_i = \sum_{i=1}^{i=n} w_i \left(R_i - B_i\right) \quad (16)$$

This equation is equivalent to Quadrant I – Quadrant II in Exhibit 8. Using this definition for selection, the sum of selection and allocation conveniently adds up to the total excess return as follows:

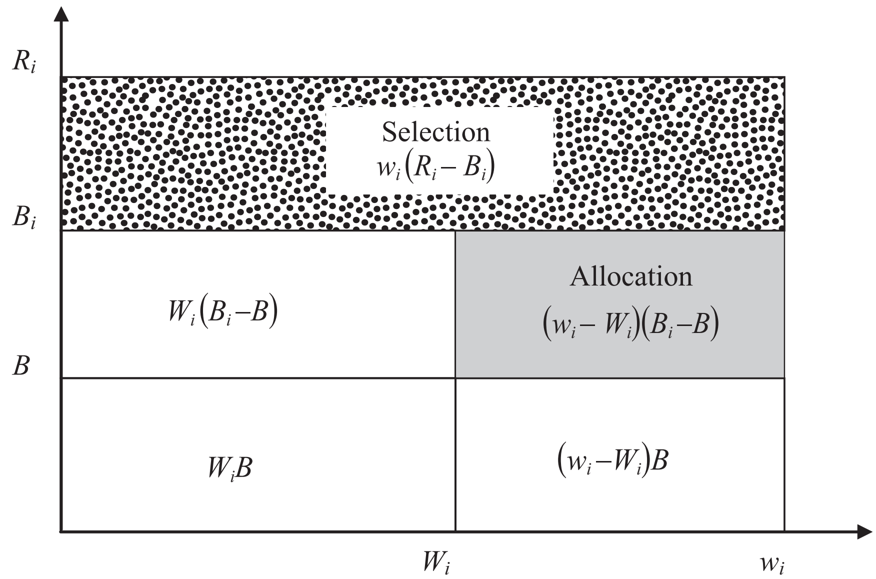
Selection + Allocation = $(R - B_S) + (B_S - B) = R - B$

The contribution to selection in the ith sector is now:

$$S_i = w_i(R_i - B_i) \quad (17)$$

The revised impact on individual sectors is graphically demonstrated in Exhibit 13. Essentially, the interaction effect is included in the stock selection effect, which is consistent with most portfolio manager's decision processes.

Exhibit 13 Interaction Effect in BF Model



The revised total selection effect, including interaction using the portfolio data from Exhibit 3, is:

$$R - B_S = 10.1\% - 6.8\% = 3.3\%$$

The selection effects in each sector are:

Energy	50% × (18% – 10%) = 4.0%
Health Care	30% × (–3.0% + 2.0%) = –0.3%
Financials	20% × (10.0% – 12.0%) = –0.4%
Total	4.0% – 0.3% – 0.4% = 3.3%

Actual portfolio weights are now used to calculate selection effects rather than calculating what would have been the contribution to selection in that sector at the benchmark weight. There is no change in the selection effect for the Energy sector because the weight in the portfolio is the same as the weight in the benchmark. The selection effect in Health Care is now –0.3%, the sum of the previous –0.2% selection effect plus –0.1% interaction. The selection effect in Financials is now –0.4%, the sum of the previous –0.6% selection effect plus 0.2% interaction. The revised results are summarized in the Exhibit 14.

Exhibit 14 BF Results of Stock Selection and Interaction Combined

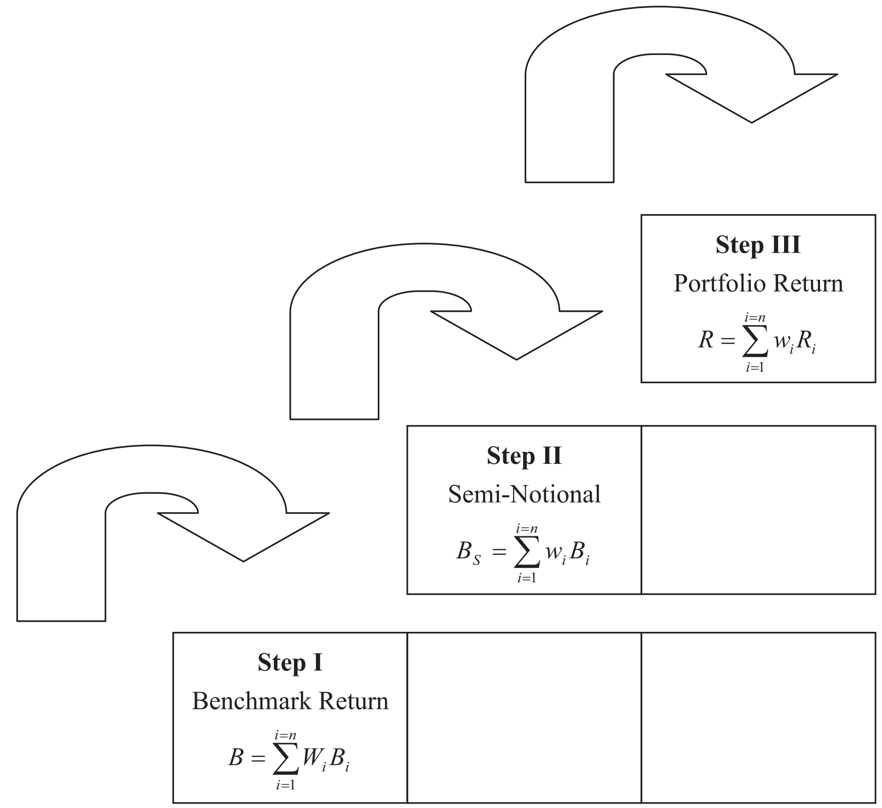
	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection
Energy	50%	50%	18%	10%	0.0%	4.0%
Health Care	30%	20%	−3%	−2%	−1.02%	−0.3%
Financials	20%	30%	10%	12%	−0.38%	−0.4%
Total	**100%**	**100%**	**10.1%**	**8.2%**	**−1.4%**	**3.3%**

In this example, in which allocation decisions are made first, interaction is appropriately included with selection. Combining selection with interaction allows the analyst to appropriately measure the impact of both the asset allocation and security selection decisions in the investment process.

Quadrant III (of Exhibit 8) of the Brinson model is now not required, which is an accurate reflection of most investment decision processes; cash is allocated to sectors and then this cash is used to purchase individual securities. Only the allocation notional fund is required to calculate return attribution, one step away from the benchmark portfolio, one step toward the final outcome of the portfolio return. Reflecting its intermediate position between the benchmark and portfolio return, the allocation notional fund is often renamed the "semi-notional fund." Rather than using the Brinson quadrants in Exhibit 8, it is preferable to think in terms of steps in the investment decision process as shown in Exhibit 15. Far more complex decision processes can be handled simply by identifying each individual step in the decision process and calculating a notional fund return corresponding to each step.

This analysis is a demonstration of an effective return attribution process. The analysis reflects the two-stage decision process of the portfolio manager: first sector allocation and then stock selection within sectors. We measure only the active decisions of the portfolio manager because they provide a complete understanding of the difference between the portfolio return and the benchmark return, and hence, provide a good explanation of the performance of the portfolio manager.

Exhibit 15 Steps in Investment Decision Process



To summarize, the Brinson model attributes excess return to allocation and selection effects. There are two versions of the Brinson model: Brinson–Hood–Beebower and Brinson–Fachler. They differ only in the calculation of individual sector allocation effects. In both versions of the Brinson model, interaction may be calculated separately or included in the selection effect.

4 GEOMETRIC EQUITY RETURN ATTRIBUTION

The Brinson models described so far quantify arithmetic excess return only. A number of geometric excess return attribution models (*geometric methods*) have also been developed, such as Allen (1991), Bain (1996), Burnie, Knowles, and Teder (1998), and Bacon (2002, 2008). In effect, these models extend the Brinson model to attribute the geometric excess return defined as:

$$\frac{(1+R)}{(1+B)} - 1 \quad (18)$$

Using the portfolio data from Exhibit 3, the geometric excess return that we want to attribute is:

$$\frac{1.101}{1.082} - 1 = 1.76\%$$

The decision to use either the arithmetic model or geometric model is primarily driven by the preference of using arithmetic or geometric excess returns.

4.1 Allocation

To identify the contribution from allocation, we can use the same intermediate or semi-notional fund used in the Brinson method. But this time we use the geometric rather than the arithmetic difference, in effect the difference between Step 1 and Step 2 of the investment decision process. Step 1 is the benchmark.

$$\frac{(1 + B_S)}{(1 + B)} - 1 \tag{19}$$

The contribution to geometric allocation in the ith sector is now

$$A_i^G = (w_i - W_i)\left(\frac{1 + B_i}{1 + B} - 1\right) \tag{20}$$

Equation 20 is analogous to Equation 15; however, the geometric difference of the sector return against the overall benchmark is used rather than the arithmetic difference. Note the total geometric allocation A^G:

$$A^G = \sum_{i=1}^{i=n} A_i^G = \frac{(1 + B_S)}{1 + B} - 1 \tag{21}$$

The geometric allocation effect for the portfolio data in Exhibit 3 is

$$\frac{(1 + B_S)}{(1 + B)} - 1 = \frac{1.068}{1.082} - 1 = -1.29\%$$

The individual sector allocation effects are

$$\text{Energy } (50\% - 50\%) \times \left(\frac{1.10}{1.082} - 1\right) = 0.0\%$$

$$\text{Health Care } (30\% - 20\%) \times \left(\frac{0.98}{1.082} - 1\right) = -0.94\%$$

$$\text{Financials } (20\% - 30\%) \times \left(\frac{1.12}{1.082} - 1\right) = -0.35\%$$

Total: 0.0% – 0.94% – 0.35% = –1.29%

Given that the benchmark return is positive, the geometric excess return is a smaller negative number than the arithmetic excess return of –1.4%. The contributions to allocation are similar but slightly less than the arithmetic allocation. For comparison, the arithmetic effects are Energy 0.0%, Health Care –1.02%, and Financials –0.38%. With either method, the sign will always be the same.

EXAMPLE 8

Geometric Allocation: An Illustration

Exhibit 16 Sample Portfolio Data

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Technology	20%	30%	−11.0%	−10.0%
Telecommunications	30%	40%	−5.0%	−8.0%

(continued)

Exhibit 16 (Continued)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Utilities	50%	30%	−8.0%	−5.0%
Total	**100%**	**100%**	**−7.7%**	**−7.7%**

Using the BF method, the geometric allocation effect of Utilities based on the portfolio data in Exhibit 16 is:

A 0.54%

B 0.59%

C 0.88%

Solution:

B is correct. 1 + B is calculated as 1 − 7.7% = 1 − 0.077 = 0.923 and 1 + B_i as 1 − 5.0% =1 − 0.05 = 0.95. Thus,

$$\left(w_i - W_i\right)\left(\frac{1 + B_i}{1 + B} - 1\right) = \left(50\% - 30\%\right) \times \left(\frac{0.95}{0.923} - 1\right) = 0.59\%$$

The portfolio was 20% overweight in a sector geometrically outperforming the overall benchmark by 2.93%, therefore contributing 0.59% to the geometric excess return.

A is incorrect: $W_i\left(\frac{1 + B_i}{1 + B} - 1\right) = 30\% \times \left(\frac{0.95}{0.923} - 1\right) = 0.88\%$. The benchmark weight of 30% has been used incorrectly rather than the 20% overweight position.

C is incorrect: $(w_i - W_i)(B_i - B)$ = (50% − 30%) × (−5.0% + 7.7%) = 0.54. This is an arithmetic calculation.

4.2 Selection

To identify the contribution from selection, we can use the ratio of the portfolio return compared with the same intermediate or semi-notional return, effectively the difference between Step 2 and Step 3 of the decision process.

$$\frac{\left(1 + R\right)}{\left(1 + B_S\right)} - 1 \quad (22)$$

This approach implicitly includes interaction in the selection effect, which is in line with the decision process of most portfolio managers.[5]

The contribution of the ith sector to geometric selection is now

$$S_i^G = w_i\left(\frac{1 + R_i}{1 + B_i} - 1\right)\frac{\left(1 + B_i\right)}{\left(1 + B_S\right)} \quad (23)$$

Equation 23 can be compared with its arithmetic counterpart, Equation 9. In Equation 23, we use the geometric excess return between the portfolio sector return and the benchmark sector return rather than the arithmetic excess return. The

5 Although it is possible to calculate a geometric interaction effect this is rarely done in practice.

adjustment factor $\frac{1+B_i}{1+B_S}$ reflects the fact that equivalent geometric excess returns will add different value to the overall portfolio depending on the benchmark return in the sector. In short, outperformance in a rising market will contribute more excess return to the overall portfolio than equivalent outperformance in a falling market.

Note the total geometric selection S^G:

$$S^G = \sum_{i=1}^{i=n} S_i^G = \frac{(1+R)}{(1+B_S)} - 1 \quad (24)$$

The contribution to geometric excess return from selection is

$$\frac{(1+R)}{(1+B_S)} - 1 = \frac{1.101}{1.068} - 1 = 3.09\%$$

Individual sector selection effects are

$$\text{Energy } 50\% \times \left(\frac{1.18}{1.10} - 1\right) \times \frac{1.10}{1.068} = 3.75\%$$

$$\text{Health Care } 30\% \times \left(\frac{0.97}{0.98} - 1\right) \times \frac{0.98}{1.068} = -0.28\%$$

$$\text{Financials } 20\% \times \left(\frac{1.10}{1.12} - 1\right) \times \frac{1.12}{1.068} = -0.37\%$$

Total: 3.75% – 0.28% – 0.37% = 3.09%

Again, as expected, the geometric selection effects are similar to the arithmetic selection effects but slightly smaller in magnitude. The geometric return attribution effects are summarized in Exhibit 17:

Exhibit 17 Geometric Return Attribution with Stock Selection and Interaction Combined

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection
Energy	50%	50%	18.0%	10%	0.0%	3.75%
Health Care	30%	20%	−3%	−2%	−0.94%	−0.28%
Financials	20%	30%	10%	12%	−0.35%	−0.37%
Total	**100%**	**100%**	**10.1%**	**8.2%**	**−1.29%**	**3.09%**

EXAMPLE 9

Geometric Selection

Exhibit 18 Sample Portfolio Data

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Materials	30%	20%	12.0%	8.0%
Industrials	30%	20%	8.0%	10.0%

(continued)

Exhibit 18 (Continued)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Consumer Staples	40%	60%	6.0%	8.0%
Total	**100%**	**100%**	**8.4%**	**8.4%**

Using the BF method, the selection effect including interaction of Industrials based on the portfolio data in Exhibit 18 is:

A –0.60%

B –0.55%

C –0.40%

Solution:

B is correct: $w_i\left(\frac{1+R_i}{1+B_i}-1\right)\frac{(1+B_i)}{(1+B_S)} = 30\% \times \left(\frac{1.08}{1.1}-1\right) \times \frac{1.1}{1.086} = –0.55\%.$

Note that B_S = (30% × 8%) + (30% × 10%) + (40% × 8%) = 8.6%. In the industrials sector, the portfolio manager underperformed by 1.81% geometrically. The overall contribution from selection in this sector is thus –0.55%.

A is incorrect: $w_i(R_i – B_i)$ = 30% × (8.0% – 10.0%) = –0.6%. This is an arithmetic calculation.

C is incorrect: $W_i(R_i – B_i)$ = 20% × (8.0% – 10.0%) = –0.4%. This is an arithmetic calculation using the benchmark weight.

The total selection and allocation effects compound together to produce the geometric excess return:

$$\frac{(1+R)}{(1+B_S)}\frac{(1+B_S)}{(1+B)}-1=\frac{(1+R)}{(1+B)}-1$$

or

$$\left(1+S^G\right)\left(1+A^G\right)-1=\frac{(1+R)}{(1+B)}-1$$

For example, based on the data in Exhibit 3 from which we computed R = 0.101, B_S = 0.068, and B = 0.082:

$$\frac{1.101}{1.068}\times\frac{1.068}{1.082}-1=\frac{1.101}{1.082}-1=(1.0309)\times(1-0.0129)-1=1.76\%$$

5 HOLDINGS-BASED AND TRANSACTION-BASED RETURN ATTRIBUTION

We can distinguish between three types of attribution characterized by the information used to calculate attribution effects: returns -based, holdings-based, and transaction-based attribution. The types are listed in order of increasing information requirements. However, holdings-based and transaction-based attribution are the common types used in current practice.

5.1 Returns-Based Attribution

Returns-based (or factor) attribution uses only the total portfolio returns over a period of time to identify the factors that have generated the returns observed. Returns-based attribution is obviously most appropriate when the underlying portfolio holding information is unavailable, or not available with sufficient frequency, at the required level of detail. An example is hedge funds because these vehicles are not transparent in disclosure of holdings. This type of attribution is the quickest and easiest to implement, but because it does not access the underlying holdings, it is the least accurate.

5.2 Holdings-Based Attribution

Holdings-based attribution is calculated by reference to the underlying beginning-period holdings of the portfolio only. Typically, holdings-based attribution is calculated using monthly, weekly, or daily data, with shorter time periods leading to greater accuracy. For longer evaluation periods, the attribution results for the shorter measurement periods are linked together.

Holdings-based attribution fails to capture the impact of any transactions made between measurement periods and, therefore, will not reconcile to the actual portfolio return. The residual caused by ignoring transactions might be described as a timing or trading effect. Holdings-based analysis is appropriate for passive investment strategies with little turnover. Valuing the portfolio with the same prices used to calculate the underlying benchmark index will remove one potential difference between the portfolio and benchmark returns that is not a management effect.

5.3 Transaction-Based Attribution

Transaction-based attribution is calculated by using both the holdings of the portfolio and the transactions (purchases and sales) that occurred during the evaluation period. The difference between transaction-based and holdings-based attribution is the calculation of inputs to the attribution analysis. For holdings-based attribution, transactions are ignored and weights are calculated using the beginning period position exclusively. The returns are simply those returns that would have been achieved, including income and corporate actions (e.g., stock splits) but excluding any sales and purchases, if the beginning period holdings had been held to the end of the period.

For transaction-based attribution, both the weights and returns reflect all transactions during the period, including transaction costs. Transaction-based attribution is the most accurate type of attribution analysis. This type of attribution is also the most difficult and time consuming to implement. To obtain meaningful results, the underlying data must be complete, accurate, and reconciled from period to period. Because all the data is available, the entire excess return can be quantified and explained. The return used in the attribution analysis will reconcile with the return presented to the client, and attribution analysis can be used as a diagnostic tool to identify errors.

The choice of which type of attribution to use will very much depend on the availability and quality of the underlying data, reporting requirements for the client, and the complexity of the investment decision process. For example, in many cases, the lack of quality transaction data and/or the lack of turnover in the portfolio, may lead to daily holdings-based attribution that is just as informative as transaction-based attribution.

EXAMPLE 10

Types of Attribution

For an outside analyst, the most readily accomplished type of attribution to analyze the performance of a hedge fund is *most likely:*

A returns-based.

B holdings-based.

C transaction-based.

Solution:

A is correct. Because hedge funds generally do not disclose holdings to the public, returns -based analysis is much easier to accomplish than holdings-based or transaction-based analysis.

6 RETURN ATTRIBUTION ANALYSIS AT MULTIPLE LEVELS

The example used to demonstrate return attribution so far is a single country model in which the investment process allocates weight to industry sectors and then selects securities within each of those industry sectors. The same return attribution formulas for allocation and selection can be used at multiple levels in the entire decision process to evaluate different decisions. Consider an example in which the top level is the fund sponsor (e.g., a university endowment or a defined-benefit pension plan sponsor). At the fund sponsor level, the first decision might be to allocate a certain weight to asset classes—the strategic asset allocation. If the fund sponsor does not manage funds internally, they would delegate a second decision in the investment process to the investment managers who could decide on any tactical deviations from the strategic asset allocation. And the sponsor might choose a number of portfolio managers to manage against specific mandates within a given asset class. (Attribution at the sponsor level is sometimes called **macro attribution**; attribution at the portfolio manager level is sometimes called **micro attribution**.)

Using the investment decision process just described, the allocation formula can be used to measure the tactical asset allocation decision of the sponsor against its own strategic benchmark, which could be a weighted average of appropriate asset class indexes using the strategic asset allocation weights or a representation of the sponsor's liabilities, as in the case of many pension funds. The selection formula can be used to measure the added value of the performance of each of the portfolio managers relative to their assigned benchmarks. To the fund sponsor, the selection of portfolio managers is in effect a selection decision in the return attribution analysis performed at the highest level.

When we calculate return attribution, we have the opportunity to choose which level(s) to analyze. Consider the example of a fund sponsor who hires two investment managers for the equity portion of the fund. The overall fund has an equity allocation benchmark of:

50% Large-Cap Value Equities

25% Small-Cap Value Equities

25% Large-Cap Growth Equities

For this overall allocation, the fund sponsor hires Value Portfolio Manager to manage the large-cap and small-cap value allocations and Growth Portfolio Manager to manage the growth equity allocation. For a specific time period, this arrangement results in the performance shown in Exhibit 19.

Exhibit 19 Performance of Value and Growth Equity Managers

	Fund Weight	Fund Return	Benchmark Weight	Benchmark Return
Total	**100%**	**0.95**	**100%**	**–0.03**
Value Portfolio Manager	78%	0.99	75%	0.32
Small-cap value equities	20%	2.39	25%	1.52
Large-cap value equities	58%	0.51	50%	–0.28
Growth Portfolio Manager	22%	0.82	25%	–1.08
Large-cap growth equities	22%	0.82	25%	–1.08

You will note that this fund outperformed its total benchmark by 98 bps (95 bps compared with –3 bps). You will also note that the fund sponsor has allocated 78% of the total fund to the Value Portfolio Manager, exceeding the benchmark weight of 75%, with the remaining 22% allocated to the Growth Portfolio Manager, below the benchmark 25%. This overall weighting decision is that of the fund sponsor.

Within each of these total allocations, the portfolio manager has discretion about how to invest the assets. The managers have full discretion over the securities purchased, within the particular market segment benchmark. For this example, we assume that the Value Manager has full discretion to allocate between market cap segments—that is, how to divide the total 78% allocation to value equities between small cap and large cap.

Before we calculate our return attribution effects, we need to decide whom we want to evaluate. Do we want to evaluate the decisions of the portfolio manager, or those of the fund sponsor? In this first set of return attribution results, we will evaluate the portfolio manager's decisions. To do this, we will calculate the return attribution effects (using the BF approach) at the segment level (e.g., small-cap value, large-cap value, and large-cap growth). Using the earlier formulas, where:

$$\text{Allocation} = (w_i - W_i)(B_i - B)$$

$$\text{Selection + Interaction} = W_i(R_i - B_i) + (w_i - W_i)(R_i - B_i)$$

We can calculate the attribution effects for the small-cap value equities:

$$\text{Allocation} = (20\% - 25\%)[1.52 - (-0.03)] = -0.08$$

$$\text{Selection + Interaction} = [(25\%)(2.39 - 1.52)] + [(20\% - 25\%)(2.39 - 1.52)] = 0.17$$

Using the same approach for large-cap value equities and large-cap growth equities yields the results shown in Exhibit 20. (Note that numbers are shown to two decimal places and may not sum because of rounding.)

Exhibit 20 Segment Level Return Attribution

Return attribution (segment level)	Fund Weight	Selection + Interaction	Allocation	Total
Total	**100%**	**1.05**	**–0.07**	**0.98**
Value Portfolio Manager	78%	0.63	–0.10	0.53
Small-cap value equities	**20%**	**0.17**	**–0.08**	**0.10**
Large-cap value equities	**58%**	**0.46**	**–0.02**	**0.44**
Growth Portfolio Manager	22%	0.42	0.03	0.45
Large-cap growth equities	**22%**	**0.42**	**0.03**	**0.45**

In Exhibit 20, the attribution results in bold are calculated at the segment level. The attribution results at the next level above, the Value Portfolio Manager and Growth Portfolio Manager, are sums of the segment level results. For example, the allocation effect for the Value Portfolio Manager is equal to –0.08 + –0.02 = –0.10.

Because we are calculating the effects at the segment level and rolling up (summing) to a total for each manager, we are able to evaluate the specific decisions of the portfolio managers and their impact on the total fund. The total 98 bps of outperformance at the overall fund level is almost entirely the result of positive security selection decisions (105 bps in total). The decision of the Value Manager to underweight small cap in favor of large cap was a poor decision because the small-cap value benchmark outperformed the total benchmark (1.52% versus –0.03%) and the large-cap value benchmark underperformed the total benchmark (–0.28% versus –0.03%). Therefore, we calculate a negative allocation effect of –7 bps.

Separately, we can also evaluate the decisions of the fund sponsor. To do this, we perform a second return attribution analysis using the same set of weights and returns data in Exhibit 19, but we calculate the return attribution results at the portfolio manager level (rather than the segment level). For example, for the Value Portfolio Manager, we would calculate the effects as follows:

Allocation = (78% – 75%)[0.32 – (–0.03)] = 0.01

Selection + Interaction = [(75%)(0.99 – 0.32)] + [(78% – 75%)(0.99 – 0.32)] = 0.52

We calculate the effects in the same way for the Growth Portfolio Manager and obtain the results shown in Exhibit 21.

Exhibit 21 Return Attribution at the Portfolio Manager Level

Return Attribution (Portfolio Manager level)	Selection + Interaction	Allocation	Total
Total	0.94	0.04	0.98
Value Portfolio Manager	**0.52**	**0.01**	**0.53**
Small-cap value equities	–	–	–
Large-cap value equities	–	–	–
Growth Portfolio Manager	**0.42**	**0.03**	**0.45**
Large-cap growth equities	–	–	–

You will note that there are no return attribution effects at the segment level because we only calculated results at the manager level. Because we are measuring different decisions, the attribution results are different. There is still an overall positive security selection effect (94 bps), as we might expect. However, the total allocation effect is positive, not negative, because we are evaluating a *different* allocation decision. In this case, we are calculating the effect of the fund sponsor's decision to overweight value equities versus growth equities. We calculate a positive allocation of 4 bps, reflecting the overweighting of value equities (whose benchmark outperformed the total benchmark) and the underweighting of growth equities (whose benchmark underperformed the total benchmark).

Return attribution analysis is most often calculated with reference to the portfolio's agreed benchmark. But it is entirely possible to attribute one portfolio against another when using the same or similar investment strategy. The purpose of such analysis might be to explain an unexpected difference in return between two portfolios managed by the same portfolio manager using the same investment decision process.

Drilling down to the next level, the portfolio manager may well have a country allocation investment process.[6] At this level of analysis, the same allocation formula will calculate the impact of country allocation decisions within the manager's portfolio and the selection formula will calculate the impact of selection decisions within each country.

Drilling down another level, the portfolio manager may well have a sector allocation process, similar to that described earlier, within each country and thus the allocation formula can be used to calculate the impact of sector selection decisions within countries and the selection decisions within sectors.

Whatever the level of analysis, it is essential that the return attribution reflects the decision process of the portfolio manager. For example, a Eurozone investment strategy might use a country allocation process with security selection within each country or a sector allocation process with security selection within each industrial sector. Exhibits 22 and 23 illustrate the different analysis that might result from the same portfolio assuming a different investment process. In each case, an arithmetic Brinson and Fachler approach has been used.

Exhibit 22 Country Allocation

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection
France	20%	30%	8.0%	6.0%	0.15%	0.40%
Germany	20	35	8.0	7.0	0.07	0.20
Holland	20	10	9.0	15.0	0.76	–1.20
Italy	30	15	10.0	9.0	0.23	0.30
Spain	10	10	3.0	3.5	0.00	–0.05
Total	**100%**	**100%**	**8.30**	**7.45%**	**1.20%**	**–0.35%**

6 For some portfolios, this next level may be asset classes.

Exhibit 23 Industry Sector Allocation

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection
Energy	25%	30%	18.0%	12.0%	−0.23%	1.50%
Health Care	30	20	−3.0	−6.0	−1.35	0.90
Financial	20	30	10.0	12.0	−0.46	−0.40
Transportation	10	15	12.0	8.0	−0.03	0.40
Metals and Mining	15	5	10.0	5.0	−0.25	0.75
Total	**100%**	**100%**	**8.30%**	**7.45%**	**−2.30%**	**3.15%**

Exhibit 22 demonstrates good country allocation but negative security selection within countries, whereas Exhibit 23 demonstrates poor sector allocation but strongly positive security selection within industrial sectors. The message with regard to selection effects is completely different depending on the investment decision process used.

Drilling down to the lowest level, the same allocation and selection formulas can be used to calculate individual security decisions within sectors. For example, the allocation formula can determine the impact of over- or underweighting individual securities, whereas the selection formula will determine the contribution as a result of the returns of a security in the portfolio differing from the return of the same security in the benchmark. If the pricing sources used in the portfolio and the benchmark are identical, then any difference in return will be caused by transaction activity. Transaction activity as a result of trading expenses and bid–offer spreads will negatively affect returns, but very occasionally because of timing, the portfolio manager may be able to trade at advantageous prices during the day and recover all the transaction costs by the end of the day resulting in a positive effect.

Exhibit 24 shows the security level return attribution effects for a small portfolio of oil stocks against a customized benchmark consisting of the same oil stocks. This approach would be used by a pure stock picker, the only decisions in the portfolio being individual stock weighting and timing decisions.

Exhibit 24 Security Level Return Attribution Effects of Pure Stock Picker

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Transaction Costs and Timing Effects
Chevron Corporation	24%	30%	10%	10%	−0.18%	0.0%
Conoco Phillips	21%	25%	8%	8%	−0.04%	0.0%
ExxonMobil	41%	35%	5%	6%	−0.06%	−0.41%
Marathon Oil	6%	5%	4%	4%	−0.03%	0.0%
Newfield Exploration	8%	5%	−5%	−5%	−0.36%	0.0%
Total	**100%**	**100%**	**5.97%**	**7.05%**	**−0.67%**	**−0.41%**

The arithmetic allocation effects of each security using the BF approach are as follows:

Chevron Corporation	(24% – 30%) × (10% – 7.05%) = –0.18%
Conoco Phillips	(21% – 25%) × (8.0% – 7.05%) = –0.04%
ExxonMobil	(41% – 35%) × (6.0% – 7.05%) = –0.06%
Marathon Oil	(6% – 5%) × (4.0% – 7.05%) = –0.03%
Newfield Exploration	(8% – 5%) × (–5.0% – 7.05%) = –0.36%

Allocation in this context measures the value added from individual security selection. There are transactions in only one security during the period—ExxonMobil—therefore, there are selection effects (transaction costs and timing) in only this security. The calculation is

ExxonMobil 41% × (5.0% – 6.0%) = –0.41%

EXAMPLE 11

Levels of Return Attribution

1 At the lowest level of return attribution analysis, the selection type formula is *most likely* used to analyze:

- **A** transactions costs.
- **B** security level decisions.
- **C** individual portfolio managers.

2 At the highest level of return attribution analysis, the selection type formula is *most likely* used to analyze:

- **A** allocation decisions
- **B** security level decisions.
- **C** individual portfolio managers.

Solution to 1:

A is correct. At the lowest level, return attribution analysis is used to investigate the differences in return of a security in the portfolio and the benchmark, which is most likely caused by transaction costs. At the lowest level the allocation formula will measure individual security decision and the contribution of portfolio managers are normally measured at the highest level.

Solution to 2:

C is correct. Selection type formulas are used to evaluate the selection ability of individual portfolio managers.

FACTOR MODELS IN RETURN ATTRIBUTION

7

Return attribution is used to identify the components of excess return relative to a benchmark. As we have seen, return attribution allows us to analyze that excess return by comparing the accounting information (weights and returns) in the portfolio with those in the benchmark. The models presented thus far focus on the security selection, asset allocation, and interaction of those factors. But what if we want to control for other decisions within the investment process? Another type of return attribution uses fundamental factor models to decompose the relative contributors to excess return of a number of different factors. Fundamental factor analysis allows us to quantify the impact of specific active investment decisions within the portfolio, showing how they add or remove value relative to the benchmark.

We want to remove the effects of the market in an effort to identify the excess return generated by the active investment decisions. To do that, we return to our definition of excess return: Excess return = $R - B$.

Practitioners use many different factor models to decompose excess returns. In constructing those models, they have the ability to choose which components to attribute (which factors to use). For example, they may choose to control for investment style, currency management, or fixed-income characteristics like duration to attribute those parts of the excess return that are the result of exposures to the specific factors. As we mentioned earlier, we want to focus our analysis only on those factors that contribute to the active investment management process.

Consistent in all of these models, no matter which factors are used, there will be a portion of the excess return that is not related to any factors. This term is known as **specific asset selection**. Taking from our equation for excess return, we separate the excess return into the portion due to factors and that portion due to specific asset selection.

As an example, we will focus on a model first introduced by Fama and French (1993) that expands on the traditional capital asset pricing model (CAPM) regression, adding two additional terms for size and value. In this model, we choose to restrict our analysis to a single currency equity market. This restriction allows us to exclude a multitude of other potential investment decisions, such as currency management or country selection. Within the single equity market, the three factors include:

1. RMRF standing for ($R_M - R_F$) is the return on a market-value-weighted equity index minus the one-month T-bill rate (a proxy for the risk-free rate of return). The expected value is an equity risk premium, a factor shared with the CAPM.
2. SMB (small minus big) is the return on a portfolio of small-cap equities minus the return on a portfolio of large-cap equities. This factor captures a small-cap return premium.
3. HML (high minus low) is the return on a portfolio of high book-to-price equities minus the return on a portfolio of low book-to-price stocks. Recognizing that a high book-to-price ratio means a low price-to-book ratio (i.e., selling cheaply relative to book value), this factor represents a value return premium.[7]

The model itself is expressed as

$$R_P - R_F = \alpha + \beta_{mkt}\text{RMRF} + \beta_S\text{SMB} + \beta_V\text{HML} + \varepsilon_P$$

7 Using the book-to-price ratio avoids problems, such as division by zero, that are possible with the more familiar price-to-book ratio.

where

R_P = Portfolio Return
R_F = The return on a one-month T-bill, representing a default-risk-free return
R_M = Return on a market-value-weighted equity index
β_{mkt} = The sensitivity of the portfolio (minus risk free) returns to market (minus risk free) returns
β_S = The sensitivity of the portfolio (minus risk free) returns to SMB. A positive high number (greater than 1) suggests that the portfolio tends to hold/favor small-cap over large-cap equities.
β_V = The sensitivity of the portfolio (minus risk free) returns to HML. A positive high number (greater than 1) suggests that the portfolio tends to hold/favor value-oriented versus growth-oriented equities.
ε_P = Error term

By analyzing the results of a factor return attribution analysis, we can infer the relative strengths and/or weaknesses of the investment decisions. For example, using the Fama–French model provided earlier, we obtain the following results from the return attribution analysis.

Exhibit 25 Fama–French Factor Model

Factor	Portfolio Sensitivity	Benchmark Sensitivity	Difference	Factor Return	Absolute Contribution	Proportional Contribution
$R_M - R_F$	1.05	1.09	–0.04	6.88	–0.2752	29%
SMB	1.2	1.1	0.1	–3.82	–0.3820	40%
HML	0.3	0.25	0.05	–4.97	–0.2485	26%
				Return attributed to factors	–0.9057	96%
				Return attributed to specific asset selection	–0.0400	4%
				Excess return	–0.9457	100%

Source: Data are from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

This analysis yields some interesting results, including information about this portfolio's investment approach, how the manager generated excess return, and his or her ability to consistently add value relative to the benchmark.

First, in terms of investment approach, note that both the portfolio and the benchmark have a much higher exposure to the SMB factor (1.2 and 1.1 respectively) relative to the other two factors in our analysis. This exposure suggests that both the portfolio and benchmark tend to hold smaller capitalization equities relative to large capitalization equities. Similarly, the HML factors for both portfolio and benchmark are relatively low (0.3 and 0.25 respectively), suggesting that both portfolio and benchmark have a relatively low exposure to the value equities. Given these findings, we might conclude that this particular manager has either a small-cap growth or small-cap neutral investment style.

Second, we know that the portfolio had a negative excess return for this particular time period (–0.9457). Not surprisingly, the largest contributor to this negative excess return was the impact of the largest factor, SMB. For this time period, SMB contributed a –0.382, or 40% of the negative excess return. The decision to invest in small-cap securities, above the benchmark, clearly hurt the overall performance because the small-cap tilted portfolio (SMB) under-performed during this period. Similarly, the excess exposure to HML relative to the benchmark also negatively affected the excess return (−0.2485 or 26%). Lastly, the under-exposure to the overall market relative to the benchmark, the positive $R_M - R_F$ factor, also had a negative impact (−0.2752 or 29%).

Using this analysis, can we draw any conclusions about the investment management skill of this particular manager? The analysis reveals that 96% of the excess return can be attributed to one of the three factors: market, size, and style. The remaining 4% of the excess return not explained by those factors has a negative contribution (−0.04).

Within the factors, a higher allocation to small cap (1.2 versus 1.1 in the benchmark) and away from a growth bias (0.3 to 0.25 in the benchmark) had a negative impact. Because both small cap (−3.82 to 6.88) and value style (−4.97 to 6.88) underperformed the broad market index, the portfolio was negatively affected by those particular investment styles. We cannot say with certainty that the manager intentionally made bets in favor of either small-cap or value stocks during the investment period, but those would be reasonable questions that an evaluator might pose to the portfolio manager.

EXAMPLE 12

Factor Models

Using the return attribution analysis in Exhibit 25, select the best answer to the following:

1 About how much of the excess return is explained by the HML factor?

- **A** 29%
- **B** 26%
- **C** 96%

2 By how much did the portfolio overweight HML, relative to the benchmark?

- **A** 0.10
- **B** 0.05
- **C** 0.09

3 The impact on the excess return of the decision to overweight HML was:

- **A** negative because the factor return attribution analysis results indicate an absolute impact from the HML factor of −0.2485.
- **B** negative because the factor return attribution analysis results indicate an HML factor return of −4.97.
- **C** positive because the factor return attribution analysis results indicate a difference of 0.05.

Solution to 1:

B is correct. The results indicate that the HML factor contributed 26% to the overall excess return.

Solution to 2:

B is correct. The results indicate that the portfolio was 0.30, the benchmark was 0.25, and thus the difference was 0.05, or 0.30 − 0.25.

Solution to 3:

A is correct. The absolute contribution column in Exhibit 25 indicates the overall contribution to excess return from each factor. B is not correct because although the HML factor return is negative, the impact on the excess return may be positive or negative, depending on whether the portfolio is overweight or underweight HML relative to the benchmark. C is not correct because the portfolio is overweight a negative factor, which has a negative impact on the excess return.

In Example 12, using factors for market, size, and style is one of many different approaches to factor-based return attribution. Many different approaches are currently available within the industry and, relative to the asset-grouping approach—which isolates attribution effects by comparing the returns of variously constructed portfolios or groups of assets—of the Brinson or geometric models, factor models offer the following advantages:

- They are much more flexible in that they can be tailored very closely to the investment process.
- Different factor models can be used with the same portfolios to elicit return attribution effects for varying investment decisions.
- Factor models can be very sophisticated and designed to measure many effects resulting from investment management decisions.

Some of the key disadvantages relative to asset-grouping approaches include:

- Factor models are generally applied to single asset class (equity or fixed-income) portfolios, whereas asset-grouping approaches can be used for total plans or balanced portfolios containing multiple asset classes.
- Factor models are dependent on the specific factors used in the analysis. There are clear limits to every factor model, which is why some practitioners will apply several different models.
- If the factor model is particularly complex, for instance some fixed-income factor models, the data and computer software costs can be significant.
- In general, fewer varieties of asset-grouping models are used in the industry, making asset-grouping analysis results much more recognizable. Practitioners may encounter a large variety of factor models in the industry.
- Asset-grouping models are often easier to understand in terms of investment decisions.

So, although factor models may be more complex and difficult to use, their return attribution results can be used together with some of the asset-grouping approaches to provide a more complete understanding of the merits of the investment process.

EXAMPLE 13

Advantages and Disadvantages of Factor Models

1 Which of the following is an advantage of factor models relative to an asset-grouping approach?

   A They can be tailored to very subtle investment decisions.

   B They are dependent on the specific factors used in the analysis.

   C A very limited variety of factor models exists in the industry.

2 Which models are more frequently used with balanced portfolios?

   A Factor models because they are easily configured to consider multiple asset classes within one model.

B Asset-grouping models because the data requirements are less.

C Asset-grouping models because they can be used with multiple asset classes.

Solution to 1:

A is correct. Factor models can be designed to take into account any number of factors, and can thus be constructed to analyze very small (subtle) investment decisions. B is not correct because dependency on the specific factors is a disadvantage relative to asset-grouping models. C is not correct because a very large variety of factor model approaches are common throughout the industry.

Solution to 2:

C is correct. Asset-grouping models are most commonly used with balanced or multi-asset-class portfolios. A is not correct because factor models are generally not used because the models are usually more difficult to apply to multiple asset classes. B is not correct because the data requirements do not make it easier to use asset-grouping models for multiple asset classes.

8 FIXED-INCOME RETURN ATTRIBUTION: AN OVERVIEW

Investing in fixed-income securities is different from investing in equities because different factors drive bond returns. Interest rate risk is the primary risk faced by fixed-income securities (bond prices generally move inversely to interest rate changes).[8] The risk of non-payment of promised interest and/or principal (default risk) is a secondary risk for fixed-income investors. For equity investors, company-specific differences are important because, for example, they can lead one company to overtake a competitor and lead to very different returns for investors in those issues. Fixed-income investors have a limited upside defined by promised interest and principal (at maturity) payments and are generally less concerned with individual issuer selection within a given credit rating band, especially for "high-grade" bonds. For example, investors in "speculative-grade" bonds pay more attention to individual issuer differences. The decisions in fixed-income investing generally revolve around the desired levels of interest rate risk (sensitivity, often summarized by a measure known as *duration*) and credit risk (which may depend on the extra expected return the marketplace is offering for bearing more risk than a government bond). As a consequence, fixed-income investment nearly always involves decisions relative to sensitivity to interest rate changes and credit quality. Exchange rate exposures (for multi-currency portfolios) are an example of other decisions that may be relevant.

The Brinson approach was designed chiefly with equity return attribution in mind and is suitable for only the simplest of fixed-income investment processes. Exhibit 26 shows hypothetical output from applying a Brinson approach to fixed-income attribution. Fixed-income indexes that can serve as benchmarks are readily available and the portfolio and benchmark can be segmented by various fixed-income categories. For instance, if we want to understand the impact of investment decisions across the yield curve, we could choose to segment the portfolio and benchmark by maturity bands of increasing length. This choice might provide a return attribution analysis as show in Exhibit 26.

8 If bond investors expect interest rates to increase, they will lower the interest rate exposure in their portfolios, such as by shortening maturities. Conversely, if they expect interest rates to fall, they will increase interest rate risk, such as by lengthening maturities.

Exhibit 26 Return Attribution Analysis by Maturity Band, USD Portfolio

Maturity	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Selection and Interaction	Allocation
0–1 Year	20%	20%	0.15%	0.14%	0.00%	0.00%
1–3 Years	50%	30%	1.65%	1.55%	0.05%	–0.08%
4–9 Years	20%	30%	2.80%	2.75%	0.01%	–0.08%
10+ Years	10%	20%	3.35%	3.25%	0.01%	–0.13%
Total	**100%**	**100%**	**1.75%**	**1.97%**	**0.07%**	**–0.29%**

If we assume that the decision to invest in particular parts of the yield curve is a separate and distinct decision, then we could use the return attribution analysis to evaluate that yield curve weighting decision. With these results, we clearly see an overweighting along the shorter segments of the yield curve, with 1–3 years overweight (50% versus 30%), and both 4–9 and 10+ years underweight. The manager may have anticipated an increase in interest rates across maturities, when being underweight long maturities would tend to limit losses (because long bonds are typically more interest rate sensitive than short bonds); but the manager was incorrect. This analysis suggests that these maturity allocation decisions account for the under performance of the portfolio during this period. The allocation category essentially tries to capture interest rate exposure decisions (yield curve positioning), whereas selection and interaction cover all other decisions.

Although this Brinson methodology is somewhat useful, it only measures one allocation decision. In the Exhibit 26 example, we measured only allocation to different parts of the yield curve. However, most fixed-income decision processes include multiple decisions relative to the benchmark and in those cases a fixed-income attribution model designed to evaluate those multiple decisions is appropriate. For illustration, we will consider a hypothetical model based on two primary decisions: yield curve positioning and the credit spread (each of which is further analyzed into component decisions).

The most common measurement of a portfolio's interest rate risk is its duration. We define **duration** as the percentage change in bond price based on an unanticipated small change in interest rates (in particular, the market-required yield to maturity). In the absence of other factors, a higher duration means a higher sensitivity to changes in interest rates. **Convexity** is another interest rate risk measure that may be used in conjunction with duration. Broadly speaking, convexity captures the degree of nonlinearity (curvature) in the relationship between price change and yield change.

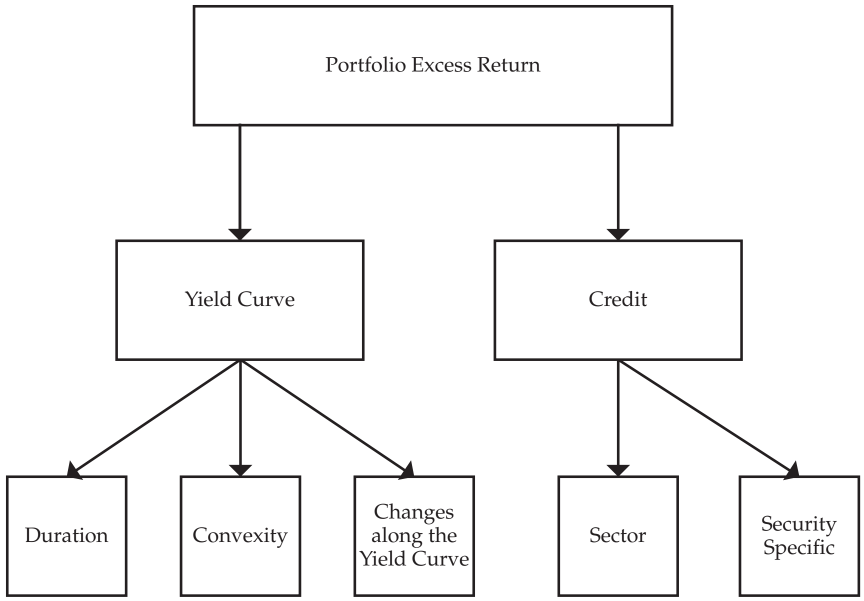
If the credit risk within a specific security is higher than that of a government bond, the debt issuer will need to pay a premium over the current government bond yield. That premium, known as the **credit spread**, will be higher for higher default-risk securities. The spread will change over time as the security's credit risk changes and as risk aversion in the marketplace increases and decreases.

The credit spread of any given security can be thought of as having two primary sources. The first source is related to industry **sector** (e.g., technology) because the market conditions in a sector affect all companies operating within it. The remaining portion of the credit risk that is not related to sector is defined as **security specific**.

In addition to the choice of interest rate sensitivity, fixed-income investment managers may also make investment decisions about the credit risk in their portfolios. They will look to add value to those portfolios, relative to their benchmarks, by anticipating the changes in those credit spreads. For example, if a manager expects the credit spreads for certain sectors to contract over time, that manager may choose to overweight those sectors in search of additional outperformance. When constructing our return attribution analysis, we want to be able to consider the relative impact of all these decisions. By considering more than one factor, multi-factor fixed-income return attribution models allow us to evaluate the merits of these multiple decisions on the portfolio.

Consider, for example, the decomposition of portfolio excess return shown in Exhibit 27.

Exhibit 27 An Example of a Fixed Income Return Attribution Model



In this example, the excess portfolio return is divided into two primary decision areas: Yield curve positioning (or interest rate management) and credit positioning. The yield curve is then further broken down into the changes as a result of duration, convexity, and changes along the yield curve. With an upward sloping yield curve, there is generally a positive return to the passage of time, all else being equal. This element of return is sometimes called the roll-down. Credit is divided into the excess return generated by the sector bets and the security specific bets.

As a means of understanding fixed-income return attribution, we have included sample results for two different fixed-income portfolios, both managed relative to the same fixed-income benchmark. Fixed-income return attribution models tend to be data and computationally intensive and are typically calculated by one of many software products available in the marketplace. For our purposes, we will focus on understanding and interpreting the results.

In this example, we can compare the fixed-income return attribution results for two portfolio managers, Portfolio One and Portfolio Two. Portfolio One's manager claims an investment style that constructs portfolios to be very sensitive to interest rate changes. The manager invests heavily in specific economic models designed to identify the most under-valued economic sectors. The manager also acknowledges doing less research on specific fixed-income securities. Portfolio Two's manager focuses investment research primarily on predicting the relative changes in economic sectors and looks for specific bonds within those sectors. The manager does not look to add value through yield curve positioning relative to the benchmark.

Our analysis yields the results shown in Exhibit 28.

Exhibit 28 Fixed-Income Return Attribution Results

	Portfolio One	Portfolio Two
Total portfolio return	2.35	1.89
Total benchmark return	1.86	1.86
Total excess return	**0.49**	**0.03**
Yield curve total	**0.41**	**–0.17**
Duration	0.31	–0.25
Convexity	0.02	0.03
Changes along yield curve	0.08	0.05
Credit	**0.08**	**0.20**
Sector	0.29	0.11
Security specific	–0.21	0.09

In reviewing the results, we see that Portfolio One did manage to achieve a higher excess return than Portfolio Two over the same time period. That higher excess return appears to be the result primarily of the interest rate bets of Portfolio One over the investment period. Looking at a total of 49 bps of excess return, 41 bps were generated through those interest rate movement bets. Additionally, Portfolio One benefits from sector bets in relation to the benchmark, providing 29 bps of additional excess return. Portfolio Two also generated some positive excess return from both sector and security investment decisions, 20 bps, but it was mostly offset by losing 17 bps from yield curve positioning relative to the benchmark.

Considering our evaluation of each manager's investment strategies, we might infer the following. The claim that Portfolio One constructs portfolios that are positioned well to interest rate changes certainly appears true for this time period. For Portfolio Two, the decision to be consistent with the interest rate sensitivities of the benchmark may not be working as well as he or she hoped. Furthermore, the negative performance from interest rate sensitivity detracts from what might be an otherwise successful ability to identify well-performing securities within their benchmark sectors.

As with all return attribution analysis, we should remember that these fixed-income return attribution results help inform our understanding of the investment process for the particular measurement period. We need to combine these results with what we know to be true about the actual investment process through both qualitative analysis and additional quantitative data. And we may also want to extend our analysis to additional time periods or possibly even consider other factors.

When analyzing the fixed-income investment management process, practitioners may find that the standard Brinson return attribution models provide an adequate level of analysis. In most cases, however, they are likely to prefer the precision of a specialized, multi-factor approach that can be more closely tailored to the actual investment process. They will then have to decide whether those benefits from a specialized fixed-income return attribution approach will be worth the incremental costs, including specialized software models and much more in-depth data requirements.

EXAMPLE 14

Fixed-Income Attribution

1 Why might a specialized fixed-income return attribution analysis be preferable to a standard Brinson approach?

A Specialized fixed-income return attribution models are easier to implement than standard Brinson approach.

B Fixed-income return attribution models can be tailored to more closely match the fixed-income investment process.

C Fixed-income return attribution models have less specialized data and software requirements.

2 For Portfolio One in Exhibit 28, what is the most important contributor to excess return?

A The positioning of the portfolio's sensitivity to interest rates relative to the benchmark.

B The selection of securities within the portfolio versus the benchmark.

C The sector weighting of the portfolio relative to the benchmark.

3 For Portfolio Two in Exhibit 28, which factor most negatively affected the excess return?

A The positioning of the portfolio's sensitivity to interest rates relative to the benchmark.

B The selection of securities within the portfolio versus the benchmark.

C The sector weighting of the portfolio relative to the benchmark.

4 In Exhibit 28, what investment decision of Portfolio Two might be superior to that of Portfolio One?

A Sector bets

B Interest rate bets

C Security selection

Solution to 1:

B is correct. Fixed-income return attribution models can be closely tailored to match the investment process. A is not correct because fixed-income return attribution models are not typically easier to implement than the Brinson approach. C is not correct because fixed-income return attribution models typically require more data and sophisticated software.

Solution to 2:

A is correct. Portfolio One achieved 41 bps from the impact of yield curve positioning and 31 bps specifically from interest rate sensitivity (duration), both of which are higher than sector weighting (29 bps) and security selection (−21 bps).

Solution to 3:

A is correct. Portfolio Two lost 17 bps from the impact of yield curve positioning The −17 bps were offset by the positive sector weighting (11 bps) and security selection (9 bps).

Solution to 4:

C is correct. Portfolio Two gained 9 bps from security selection relative to −21 bps for Portfolio One. A is not correct because Portfolio Two gained only 11 bps relative to Portfolio One, which gained 29 bps from sector positioning. B is not correct because Portfolio One gained 41 bps relative to Portfolio Two, which lost 17 bps.

PROBLEMS IN RETURN ATTRIBUTION ANALYSIS

9

With such a key role in the analysis of the investment decision process, any anomalies revealed in the calculation of return attribution will inevitably attract a significant amount of attention. Considerable effort may be required to ensure the return attribution analysis genuinely reflects the decision process and that the data is of sufficient quality. Residuals of any size will naturally bring into question the accuracy of the return attribution analysis. A new step in the decision process, a new instrument, or a new strategy may require the redesign of the return attribution analysis to provide meaningful information.

9.1 Data Input

Clearly the quality of the return attribution analysis is only as good as the data input. The only inputs required for the Brinson model are portfolio weights and returns and benchmark weights and returns. If the difference between total portfolio return and total benchmark return is to be completely explained, then the sum of each sector weight and return for both the portfolio and benchmark, as described in Equations 1 and 2, must equal the total returns presented to the client.

For holdings-based return attribution the weight used is the beginning period weight for the sector expressed as a percentage of the total portfolio beginning period weight.

For example, the weights for the portfolio shown in Exhibit 3 were calculated for use in a holdings-based analysis as shown in Exhibit 29.

Exhibit 29 Holdings-Based Beginning Weights

	Beginning Market Value (millions)	Percentage Weight
Energy	$500	50%
Health Care	$300	30%
Financials	$200	20%
Total	**$1,000**	**100%**

The end market values for each sector and the returns used in the return attribution analysis are shown in Exhibit 30.

Exhibit 30 Holdings-Based Sector Returns

Sector	End Market Value (millions)	Return
Energy	$590	$\frac{590}{500} - 1 = 18.0\%$
Healthcare	$291	$\frac{291}{300} - 1 = -3.0\%$
Financials	$220	$\frac{220}{200} - 1 = 10.0\%$
Total	$1,101	$\frac{1101}{1000} - 1 = 10.1\%$

For transaction-based return attribution, the weight used must reflect the impact of transactions during the period and must be consistent with the underlying return calculation used. For example, if the calculation method used for the period is modified Dietz then the denominator in the modified Dietz calculation provides an appropriate weight for use in return attribution analysis.

The period of analysis for the portfolio data in Exhibit 3 is a 30-day month. Although ignored in the monthly holding analysis used previously, there are transactions at the end of the 10th day, namely a $5 million outflow out of the health care sector into the financials sector. Using a transaction-based approach, the weights must now be adjusted using the denominator of the modified Dietz method of calculation as shown in Exhibit 31.

Exhibit 31 Transaction-Based Weights

Sector	Modified Dietz Denominator ($ millions)	Percentage Weight
Energy	$500	50%
Health Care	$300 – (5 × 20/30) = $296.67	29.67%
Financials	$200 + (5 × 20/30) = $203.33	20.33%
Total	**$1,000**	**100%**

It is very important to use a consistent return methodology for each sector and the total portfolio return within the attribution analysis. For example, when a time-weighted return is used for one sector with a large cash flow and a modified Dietz return is used for all the other sectors, then the sum of sector weights and returns is unlikely to add up to the total return, which will lead to a residual in the attribution analysis.

9.2 Residuals

Residuals in return attribution can be caused by a number of reasons.

9.2.1 *Holdings-Based Return Attribution*

Because holdings-based return attribution naturally ignores transactions in the period, there will be a reconciliation difference or residual between the return implicitly used in the return attribution analysis and the return presented to the client.

9.2.2 *Errors in Portfolio and Benchmark Returns*

Any failure of the sum product of the weights and returns included in the return attribution of either the portfolio or benchmark to add up to the reported total portfolio or benchmark return will cause a residual.

Among the possible causes are (this list is not exhaustive):

- the failure to include all investment categories in the portfolio in the return attribution analysis;
- the failure to include all investment instruments, such as forward currency contracts or other derivative instruments; or
- mixing the return methodologies used to calculate sector returns and the total return.

In the benchmark, a residual might be caused by an error in the calculation of a customized benchmark, it might be an error caused by the index provider, or more likely it might be an unexpected transaction in a constituent security in the index, such as a corporate action.

9.2.3 *Incomplete Models*

The model itself may be incomplete and not identifying all the impacts within the investment decision process. One may need to add more factors to the model or try a different combination of factors that better explain the investment process.

9.2.4 *Multi-period Analysis*

Although multi-period analysis is beyond the scope of this reading, it should be noted that arithmetic excess returns naturally do not add up over multiple periods and, therefore, arithmetic return attribution analysis will also not normally add up. This characteristic will lead to a residual unless a geometric attribution methodology is used or more likely a smoothing algorithm is applied to the residual, redistributing the residual to other terms within the return attribution analysis.

EXAMPLE 15

Residuals

Residuals in return attribution analysis are *most likely* caused by:

A the interaction between investment decisions.

B portfolio transactions within the evaluation period.

C investing in securities not included in the benchmark.

Solution:

B is correct. The interaction between investment decisions is not a residual, but rather it is a directly calculable effect, so A is not correct. The return attribution of securities not included in the benchmark will depend on the investment process and should not lead to a residual.

9.3 Off-Benchmark Decisions

From time to time, if allowed by their investment mandate, portfolio managers may invest in securities not included in their benchmark (**off-benchmark decisions** or **zero-weight benchmark decisions**). The return attribution of these decisions depends on the rationale behind the decision in the first place.

For example, if an investment committee makes the decision to invest outside of the benchmark to specifically gain exposure to the transportation sector, a sector allocation decision has clearly been made and should be measured as such. If the benchmark does not include the transportation sector, its weight in the benchmark index will be zero and any allocation to transportation will thus be overweight. To measure the impact of this overweight decision, a suitable transportation sector index must be chosen.[9] This will not affect the overall benchmark return because the weight is zero. The portfolio manager will then invest in transportation securities to implement this sector decision; the selection impact of these securities is measured in the normal way.

But if the decision to invest in the transportation sector is solely the desire of the portfolio manager to purchase an individual security and allocation to the transportation sector did not feature in the investment process, then it would be inappropriate to measure the impact of being overweight the transportation sector. In this case, it would be more appropriate to measure the performance of the security against the overall benchmark thus ensuring its entire contribution to excess return, positive or negative, is always a selection effect.

A decision may be allocation driven but it may be impossible to identify an appropriate sector index. In such circumstances, the best proxy for the sector index return may well be the actual return achieved by the portfolio manager in that sector. This approach would result in the entire impact being allocated to allocation with no selection effect. In virtually all circumstances, it would be inappropriate to use a benchmark return of zero.

Three alternative presentations of a portfolio containing an off-benchmark decision are shown in Exhibit 32, Exhibit 33, and Exhibit 34.

Exhibit 32 Portfolio with an Off-Benchmark Decision (Scenario 1)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection
Energy	50%	50%	18%	10%	0.0%	4.0%
Health Care	30%	20%	–3%	–2%	–1.02%	–0.3%
Financials	15%	30%	10%	12%	–0.57%	–0.3%
Transportation	5%	0%	12%	4%	–0.21%	0.4%
Total	**100%**	**100%**	**10.2%**	**8.2%**	**–1.8%**	**+3.8%**

The arithmetic return attribution effects for transportation, assuming a sector allocation decision, are as follows:

Allocation: (5% – 0%) × (4% – 8.2%) = –0.21%

Selection (including interaction): 5% × (12% – 4%) = 0.4%

Alternatively, the arithmetic return attribution effects for transportation, assuming a selection only decision (note the change in benchmark for only the transportation sector) are:

9 Although the choice of sub-index will not impact the overall benchmark return because of the zero weight, this choice should be independent of the manager responsible for managing securities within the sector.

Exhibit 33 Portfolio with an Off-Benchmark Decision (Scenario 2)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection
Energy	50%	50%	18%	10%	0.0%	4.0%
Health Care	30%	20%	–3%	–2%	–1.02%	–0.3%
Financials	15%	30%	10%	12%	–0.57%	–0.3%
Transportation	5%	0%	12%	8.2%	0.0%	+0.19%
Total	**100%**	**100%**	**10.2%**	**8.2%**	**–1.59%**	**3.59%**

Allocation: (5% – 0%) × (8.2% – 8.2%) = 0.0%

Selection (including interaction): 5% × (12% – 8.2%) = 0.19%

Assuming an allocation-only decision (note the benchmark return is now identical to the portfolio return in the sector), the return attribution effects for transportation are:

Exhibit 34 Portfolio with an Off-Benchmark Decision (Scenario 3)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection
Energy	50%	50%	18%	10%	0.0%	4.0%
Health Care	30%	20%	–3%	–2%	–1.02%	–0.3%
Financials	15%	30%	10%	12%	–0.57%	–0.3%
Transportation	5%	0%	12%	12%	0.19%	0.0%
Total	**100%**	**100%**	**10.2%**	**8.2%**	**–1.4%**	**+3.4%**

Allocation: (5% – 0%) × (12.0% – 8.2%) = 0.19%

Selection (including interaction): 5% × (12% – 12%) = 0.0%

The overall effect for all three alternatives is the same—an impact of 0.19%. But in the first presentation, the fact that there has been an allocation to the transportation sector has been reflected in a cost of –0.21% offset by 0.4% contribution caused by outperformance against the transportation index. In the second presentation, the entire decision is selection driven. In the third presentation, the decision is allocation driven and the entire impact is reflected in asset allocation.

EXAMPLE 16

Off-Benchmark Decision

Off-benchmark investment decisions:

A exclusively cause allocation effects.

B may cause both allocation and selection effects.

C cannot be allocated because there is no benchmark.

Solution:

B is correct. The investment decision process will inform the method of return attribution and this may lead to both allocation and selection effects. Asset allocation effects may not result if the decision to invest outside the benchmark is an individual security driven decision.

10 CONCLUSION AND SUMMARY

Return attribution is an essential tool for all performance analysts. Return attribution is an invaluable aid to help ensure that portfolio returns are calculated correctly, facilitate a dialogue between portfolio managers and clients, generate a true understanding of the sources of added and subtracted value in the portfolio, and allow the performance analysts to participate in the investment decision process and thus add value. Among the points made in this reading are the following:

- Return attribution is an essential part of the investment decision process, identifying the sources of excess return.
- Return contribution identifies the sources of absolute return, whereas return attribution identifies the sources of relative return between the returns of the portfolio and the benchmark.
- Risk attribution complements return attribution by analyzing the risk consequences of active investment decisions, both absolute and relative to the benchmark.
- The foundation of return attribution analysis is the Brinson model formulated in the 1980s and presented in papers written by Brinson and Fachler and Brinson, Hood, and Beebower.
- The Brinson model quantifies excess return into allocation, selection and interaction effects.
- Interaction is a basic flaw of the Brinson model because it is not part of the investment decision process. The interaction effect can be removed by including it in the selection effect (in which allocation is the first decision).
- The Brinson model can be adapted for use with geometric excess returns.
- Return attribution can be calculated using returns -based, holdings-based, and transaction-based methodologies. Transaction-based attribution is the most complete and accurate method but also the most difficult to implement.
- Plan sponsors and portfolio managers make contributions to excess return at different levels in the investment decision process.
- The allocation and selection formulas can be applied at different levels in the investment decision process: sponsor level, manager level, country level, sector level, and security level.
- Another type of return attribution uses fundamental factor models to decompose relative contributors to excess return of any number of different factors.
- The Fama–French model uses factors for market, size, and style and is one of many different approaches to factor-based attribution.
- The Brinson model is adaptable for fixed income return attribution but may not be useful for that purpose if it does not correspond to the investment decision process being evaluated.

- Fixed-income managers are concerned with the sensitivity of their portfolios to the change in interest rates and spreads and they need to consider other characteristics specific to bonds, such as duration and credit risk.
- Holdings-based attribution only requires the holdings at the beginning of the period to calculate return attribution effects, assuming the portfolio manager does not undertake any transactions during the period.
- Residuals may be caused by using holdings-based analysis, errors in calculating both the portfolio or benchmark return, incomplete models, and multi-period analysis using arithmetic models.
- Any decision to invest outside the benchmark can be attributed, but only in the context of the investment decision process that led to that decision.

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PRACTICE PROBLEMS

1 Return attribution is *least* helpful for:

A identifying the sources of excess return over a benchmark.

B forecasting the excess return caused by overweighing an asset sector.

C determining whether a portfolio manager added value through security selection.

2 Return attribution is a set of techniques used to identify the sources of:

A absolute return of a portfolio.

B portfolio volatility for absolute return mandates.

C excess return of a portfolio over a benchmark return.

3 Return attribution is *best* described as a process that:

A helps a portfolio manager develop an investment strategy.

B measures the results of a portfolio manager's active decisions.

C determines the appropriateness of the portfolio manager's allocation or selection decisions.

4 Return contribution analyzes how:

A portfolio return differs from benchmark return.

B sector returns contribute to total portfolio return.

C active investment decisions contribute to total portfolio return.

5 Return contribution analysis differs from return attribution analysis in that return contribution:

A uses only the weights and returns of the portfolio, and not of the benchmark.

B explains the value added from investment decisions in comparison to the portfolio's benchmark.

C uses the Brinson model(s) to decompose returns based on allocation and security selection decisions.

6 Consider a portfolio and a benchmark containing the weights and returns given below:

	Portfolio Weight	Benchmark Weight	Sector Return	Benchmark Return
Industrial Sector	40%	50%	3.0%	2.0%
Consumer Goods Sector	60%	50%	5.0%	6.0%
Total	**100%**	**100%**	**4.2%**	**4.0%**

An analyst conducting a return contribution analysis would *most likely* conclude that:

A underweighting the industrial sector was a poor decision.

B overweighting the consumer goods sector was a good decision.

C overweighting the consumer goods sector reduced the portfolio value.

7 Risk attribution is *best* described as concerned with:

A analyzing the value added by active investment decisions.

B analyzing the volatility resulting from active investment decisions.

C decomposing the excess return into allocation and selection decisions.

8 Which of the following *best describes* an effective return attribution process? It:

A presents the historical risk of the portfolio.

B provides an understanding of portfolio excess returns.

C explains the return variability of the benchmark compared to that of the portfolio.

9 An effective attribution process *most likely*:

A ranks portfolio managers.

B reflects the investment decision process.

C attributes arithmetic excess returns but not geometric excess returns.

10 An effective return attribution process should:

A be used only internally within management.

B validate the selected benchmark as appropriate.

C reconcile to the total portfolio return presented to the client.

11 The Brinson–Fachler return attribution model improves upon the Brinson–Hood–Beebower model by:

A measuring the security selection effect relatively to the overall benchmark.

B producing a positive allocation effect for any manager who is overweight in a negative market that has outperformed the overall benchmark.

C producing a higher sum of sector contributions to allocation than the Brinson–Hood–Beebower model.

12 Consider a portfolio and a benchmark containing the weights and returns given below:

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Health Care	10%	20%	3%	–4%
Utilities	30%	30%	4%	–6%
Consumer Goods	60%	50%	–2%	–4%
Total	**100%**	**100%**	**0%**	**–5%**

The allocation effect in the consumer goods sector using the Brinson–Fachler model is:

A negative because the sector benchmark return is negative.

B negative because the sector portfolio return was negative.

C positive because the portfolio was overweight in a sector that outperformed the benchmark.

13 In the Brinson–Hood–Beebower (BHB) model, an underweight position in a sector with a negative benchmark return will generate a positive allocation effect in that sector:

A in all cases.

B only if the portfolio sector return is positive.

C only if the overall benchmark return is positive.

14 The Brinson–Hood–Beebower (BHB) model differs from the Brinson–Fachler (BF) model in how it calculates the:

A interaction effects.

B sector allocation effects.

C security selection effects.

15 Consider a portfolio and a benchmark containing the weights and returns given below:

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Energy	20%	25%	7.00%	5.00%
Health Care	25%	25%	10.00%	9.00%
Financial	30%	20%	−1.00%	2.00%
Information Technology	25%	30%	3.00%	2.00%
Total	**100%**	**100%**	**4.35%**	**4.50%**

Using the BHB model, which of the following effects is positive? The

A selection effect of Energy.

B allocation effect of Health Care.

C interaction effect of Information Technology.

16 The table below shows the excess returns for a three-sector portfolio and the returns and weights for its benchmark.

Sector	Benchmark Weight	Benchmark Return	Excess Return
Banking	30.0%	−2.0%	−2.0%
Transportation	45.0%	1.0%	3.0%
Communications	25.0%	−4.0%	−1.5%

Using the Brinson–Hood–Beebower (BHB) model, the total contribution to excess return from selection is *closest* to:

A −0.50%.

B 0.38%.

C 2.50%.

17 An equity portfolio is divided into three market capitalization segments and benchmarked against a broad market index. The segment weights and returns for a single period are as follows:

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Small Cap	40%	10%	8.5%	7.9%
Mid Cap	10%	20%	6.8%	7.3%
Large Cap	50%	70%	5.1%	5.1%
Total	**100%**	**100%**	**6.6%**	**5.8%**

Using the Brinson–Hood–Beebower (BHB) model, which of the following is *closest* to the allocation effect for the large cap segment?

A −1.0%

B 0.0%

C 0.1%

18 The exhibit below shows the weights and returns of a portfolio and its benchmark. It also shows the allocation, selection and interaction effect using the BHB model.

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection	Interaction
Consumer Discretionary	25%	25%	8.00%	6.80%	0.00%	0.30%	0.00%
Consumer Staples	35%	25%	13.00%	14.00%	1.40%	–0.25%	–0.10%
Energy	10%	25%	–9.00%	–8.60%	1.29%	–0.10%	0.06%
Health Care	30%	25%	–3.00%	–1.00%	–0.05%	–0.50%	–0.10%
Total	**100%**	**100%**	**4.75%**	**2.80%**	**2.64%**	**–0.55%**	**–0.14%**

In terms of return attribution effects, which of the following contributed most to the total portfolio performance against the benchmark?

A Allocation plus selection decisions in energy

B Selection decision in consumer discretionary

C Allocation plus selection decisions in consumer staples

19 The exhibit below shows the weights and returns of a portfolio and its benchmark.

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Financials	15%	20%	–3.50%	–4.00%
Technology	30%	35%	22.00%	19.00%
Health Care	30%	17%	–3.00%	–2.00%
Consumer Staples	25%	28%	8.00%	10.00%
Total	**100%**	**100%**	**7.18%**	**8.31%**

The selection effect of the technology segment using the Brinson–Hood–Beebower model is *closest* to:

A –0.2%.

B 0.9%.

C 1.1%

20 The exhibit below shows the weights and returns of a portfolio and its benchmark.

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Health Care	10%	20%	3%	2%
Utilities	30%	30%	4%	4%
Consumer Goods	60%	50%	7%	8%
Total	**100%**	**100%**	**5.7%**	**5.6%**

According to the Brinson–Fachler attribution model, the allocation effect from the Consumer Goods sector is *closest* to:

A –0.50%.

B 0.24%.

C 0.80%.

21 In the context of Brinson attribution models, interaction is:

A small for larger allocation bets.

B a part of the investment decision making process.

C a mathematical consequence of the manager's allocation and selection decisions.

22 The exhibit below shows the weights and returns of a portfolio and its benchmarks, as well as the geometric allocation effect.

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation Effect
Health Care	10%	20%	3%	4%	0.07%
Utilities	30%	30%	4%	3%	0.00%
Consumer Goods	60%	50%	7%	6%	0.12%
Total	**100%**	**100%**	**5.7%**	**4.7%**	**0.19%**

Assuming that security selection and interaction are combined, the geometric selection effect for the portfolio is *closest* to:

A 0.76%

B 0.81%

C 1.00%

23 The exhibit below shows the weights and returns of a portfolio and its benchmarks.

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Health Care	10%	20%	5%	7%
Utilities	30%	30%	4%	3%
Consumer Goods	60%	50%	7%	6%
Total	**100%**	**100%**	**5.9%**	**5.3%**

The geometric selection effect (including interaction) for the health care sector is *closest* to:

A –0.20%

B –0.19%

C –0.16%

24 The exhibit below shows the weights and returns of a portfolio and its benchmarks, as well as the geometric allocation and selection effects based on the Brinson–Fachler model.

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation Effect	Selection Effect
Health Care	10%	20%	5%	2%	0.22%	0.29%
Utilities	30%	30%	4%	3%	0.00%	0.29%
Consumer Goods	60%	50%	8%	6%	0.16%	1.15%
Total	**100%**	**100%**	**6.5%**	**4.3%**	**0.38%**	**1.73%**

Which of the following effects from the changes to the asset allocation shown below is correct?

A Increasing the weight of utilities would have increased that sector's selection effect.

B Increasing the weight of health care would have increased the portfolio's allocation effect.

C Decreasing the weight of consumer goods would have increased that sector's allocation effect.

25 Which return attribution method would provide the *most* accurate analysis for an active portfolio manager?

A Returns-based

B Holdings-based

C Transactions-based

26 Which of the following is *most likely* a disadvantage of transactions-based return attribution compared to a holdings-based return attribution? Transaction-based:

A requires purchase and sale data inputs into the attribution analysis.

B increases the magnitude of residuals within the attribution analysis.

C necessitates reconciliation of attribution analysis to returns presented to a client.

27 Holdings-based attribution analysis:

A can be calculated over weekly or monthly holding periods.

B is the most time consuming attribution analysis to implement.

C is most appropriate when portfolio positions are not available.

28 The sponsor of an endowment fund does not manage assets internally but rather delegates most investment decisions to portfolio managers. Which of the following decisions is *most likely* to be retained by the sponsor?

A Security selection

B Tactical asset allocation

C Strategic asset allocation

The following information relates to Questions 29–31

The sponsor of a pension plan has hired two portfolio managers for the bond portion of his fund. The bond allocation benchmark of the fund is as follows:

50% long-term government bonds (Portfolio A)

30% long-term corporate bonds (Portfolio B)

20% short-term government bonds (Portfolio C)

The sponsor hires LT portfolio manager to manage Portfolio A and Portfolio B, and hires ST portfolio manager to manage Portfolio C. In the last fiscal year, the results were as follows:

	Fund Weight	Fund Return	Benchmark Weight	Benchmark Return
LT portfolio manager				
government	55%	3.50%	50%	4.00%
corporate	30%	2.50%	30%	2.00%
Total LT bonds	**85%**	**3.15%**	**80%**	**3.25%**

(continued)

	Fund Weight	Fund Return	Benchmark Weight	Benchmark Return
ST portfolio manager	15%	2.00%	20%	1.50%
Total all bonds	**100%**	**2.98%**	**100%**	**2.90%**

29 Evaluating the decisions of the sponsor for the last fiscal year, the allocation effect of the return attribution analysis for the LT portfolio manager, using the Brinson–Fachler approach, is *closest* to:

A −0.085%.

B 0.018%.

C 0.055%.

30 Evaluating the decisions of the LT portfolio manager for the last fiscal year, the allocation effect of the return attribution analysis, using the Brinson–Fachler approach, is *closest* to:

A −0.125%.

B 0.018%.

C 0.055%.

31 Assume that for the last fiscal year, with respect to LT bonds, it was observed that the allocation effect, when one evaluates the decisions of the sponsor, is less than the allocation effect, when one evaluates the decisions of the LT portfolio manager. The *most* appropriate conclusion to draw is that the:

A comparison is not valid.

B LT portfolio manager made better decisions than the sponsor.

C evaluation at the sponsor level should be completed by calculating the attribution at the segment level.

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32 The results of a multiple factor return attribution analysis are presented below.

Factor	Portfolio Sensitivity	Benchmark Sensitivity	Difference	Factor Return	Contribution
Equity risk premium	0.02	0.05	−0.03	6.71%	−0.00201
Small cap return premium	0.35	0.50	−0.15	1.96%	−0.00294
Value return premium	1.30	1.11	0.19	−0.43%	−0.00082
				Return attributed to factors	−0.00577
				Return attributed to specific asset selection	−0.00009
				Excess return	−0.00586

Which of the following statements is a valid interpretation of the results?

A Excess exposure to value stocks negatively affected the excess return.

B Excess exposure to small cap stocks negatively affected the excess return.

C The largest contributor to negative excess return was the impact of the equity risk premium factor.

33 The results of a multiple factor return attribution analysis are presented below.

Factor	Portfolio Sensitivity	Benchmark Sensitivity	Difference	Factor Return	Contribution
Equity risk premium	0.05	0.05	0.00	4.99%	0.00000
Small cap return premium	0.56	0.50	0.06	2.31%	0.00139
Value return premium	0.31	1.21	−0.90	1.01%	−0.00909
				Return attributed to factors	−0.00770
				Return attributed to specific asset selection	−0.00009
				Excess return	−0.00779

The result that *best* shows that the impact on excess return of the decision to overweight the small-cap return premium factor was positive is the:

A small-cap factor return of 2.31%.

B contribution to excess return of 0.00139.

C difference in sensitivity of 0.06 for the small cap return premium.

34 The results of a multiple factor return attribution analysis are presented below.

Factor	Portfolio Sensitivity	Benchmark Sensitivity	Difference	Factor Return	Contribution
Equity risk premium	0.05	0.07	−0.02	3.04%	−0.00061
Small cap return premium	1.23	0.98	0.25	0.44%	0.00110
Value return premium	1.17	0.64	0.53	2.76%	0.01463
				Return attributed to factors	0.01512
				Return attributed to specific asset selection	−0.00009
				Excess return	0.01503

Based on the above results, which of the following conclusions is *most* appropriate? The portfolio manager employs a:

A passive investment approach.

B small-cap value investment style.

C large-cap growth investment style.

35 Compared to asset-grouping models of attribution, factor models:

A can be tailored more closely to the investment process.

B are easier to understand in terms of investment decisions.

C can be applied more directly to balanced portfolios containing multiple asset classes.

36 Comparing asset-grouping and factor models of attribution, factor models are:

A independent of the model specification.

B available in more varieties in the industry.

C relatively inexpensive to use due to computer advances.

37 Data and implementation costs aside, using a Brinson attribution approach for a fixed income portfolio would *most likely* be appropriate if:

A a customized multi-factor model supports the investment decision process.

B yield curve positioning is the only important factor in the investment decision process.

C yield curve positioning, credit spread positioning, and specific bond selection are the three most important factors in the investment decision process.

38 Which of the following *best* explains why the Brinson attribution approach may not be suitable for fixed income strategies?

A Appropriate fixed income benchmarks are not readily available.

B The approach cannot analyze the impact of decisions across the yield curve.

C The approach cannot match an investment decision process based on multi-factor risk models.

39 The results of a fixed income return attribution analysis are given below.

Description	Return
Portfolio	2.15%
Benchmark	1.24%
Changes along yield curve	0.08%
Convexity	0.02%
Duration	0.31%
Sector	0.29%
Security specific	0.21%

The excess return attributable to interest rate management is *closest* to:

A 0.08%

B 0.33%

C 0.41%

40 The results of a fixed income return attribution analysis are given below.

Description	Return
Portfolio	2.15%
Benchmark	2.19%
Changes along yield curve	–0.02%
Convexity	0.03%
Duration	0.14%
Sector	0.07%
Security Specific	–0.26%

Which of the following statements *best* interprets the analysis?

A The portfolio manager's contribution to excess return due to interest rate management was 0.17%.

B The portfolio manager's sector allocation ability exceeded his skill at interest rate management.

C The portfolio manager's selection skill was the major factor in the portfolio's underperformance relative to the benchmark.

41 In which of the following scenarios will holdings based attribution lead to the *best* results?

A The use of daily holdings for a portfolio with low turnover.

B The use of daily holdings for a portfolio with high turnover.

C The use of monthly holdings for a portfolio with high turnover.

42 Residuals in return attribution would *most* likely be increased by:

A switching from an arithmetic to a geometric attribution model.

B increasing the consistency of portfolio and benchmark weights and returns.

C switching from transaction-based attribution to holdings-based attribution analysis.

43 When performing a return attribution analysis for a large cap manager, a residual effect will *most likely* occur if:

A a small cap benchmark is used.

B the analysis covers a single period.

C the analysis uses beginning of period holdings.

44 Mary Alice invested in information technology stocks, although information technology is not included in her benchmark. Data on her portfolio and her benchmark are shown below.

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Energy	25%	35%	7.0%	5.0%
Health Care	30%	35%	10.0%	9.0%
Financial	35%	30%	−1.0%	2.0%
Information Technology	10%	0%	3.0%	2.0%
Total	**100%**	**100%**	**4.7%**	**5.5%**

Assuming the investment in information technology was a selection only decision, the selection (including interaction) effect is *closest* to:

A −0.35%.

B −0.25%.

C 0.10%.

45 The returns and weights of a portfolio and its benchmark for latest period are shown below. The investment committee made the decision to invest outside the benchmark specifically to gain exposure to the materials sector. To measure the impact of this decision, the committee chose a suitable materials sector index. For the period, the return on the materials sector index was 7.10%.

Sector	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Consumer Discretionary	30%	35%	6.23%	7.00%
Consumer Staples	35%	40%	2.91%	2.55%
Industrials	30%	25%	3.00%	−5.67%
Materials	5%	0%	10.80%	7.10%
Total	100%	100%	4.33%	2.05%

The allocation effect for the materials sector is *closest* to:

A −0.10%

B 0.19%

C 0.25%

46 A portfolio manager believes that a company classified in the consumer staples sector will strongly outperform in the short term. The consumer staples sector is not part of his benchmark and he purchases the stock as an individual security. The result of the return attribution process should *most likely* indicate that the:

A selection effect only is zero.

B allocation effect only is zero.

C neither the allocation nor the selection effect are zero.

SOLUTIONS

1 B is correct. Return attribution is a "backward-looking" feedback mechanism. It cannot be used to forecast future investment returns.

A is incorrect because return attribution is defined as a set of techniques used to identify the sources of excess return. C is incorrect because return attribution shows the excess return resulting from the security selection decisions of a portfolio manager.

2 C is correct. Return attribution is a set of techniques used to identify the sources of the excess return of a portfolio against its benchmark.

A is incorrect. Return contribution (also called absolute return attribution), not return attribution, analyzes the contributions of portfolio components to the total return of a portfolio. C is incorrect. Risk attribution, not return attribution, is concerned with identifying the sources of portfolio volatility (or tracking risk) for absolute (or relative) return mandates.

3 B is correct. Because return attribution utilizes a benchmark to determine excess return, the results will reflect any value added from the portfolio manager's active decisions.

A is incorrect because the portfolio manager is expected to have formulated an investment strategy prior to any return attribution calculation. C is incorrect because return attribution is used to measure the results of active investment decisions. Considerations such as client suitability are not involved.

4 B is correct. Return contribution (also called *absolute* return attribution) analyzes the contributions of portfolio components to the total return of a portfolio, with no comparison to a benchmark.

A and C are incorrect because they refer to a comparative analysis to a benchmark.

5 A is correct. Return contribution is calculated as the sum of the product of the security/sector weight multiplied by the security/sector return, without the consideration of a benchmark. Only the weights and returns of the portfolio are utilized. Return attribution would include comparison with a benchmark.

Both B and C are incorrect because they claim that in return contribution, a calculated comparison is made to a benchmark.

6 B is correct. In this exhibit, because the portfolio return in the industrial sector was less than the return in the consumer goods sector, investing more in the consumer goods sector than in the industrial sector increased the portfolio's total return.

A is incorrect because the return in the industrial sector was less than in the consumer goods sector. Thus, investing less in the industrial sector than in the consumer goods sector was a good decision. The fact that the industrial sector investment outperformed its benchmark is not relevant to return contribution analysis. C is incorrect. Because the consumer goods sector return was positive, it contributed positively to total return and thus increased the portfolio value. The fact that the consumer goods sector investment underperformed against its benchmark is not relevant to contribution analysis.

7 B is correct. Risk attribution is concerned with analyzing the risk consequences of active investment decisions.

A is incorrect. Return attribution, not risk attribution, can best be used to analyze the value added by active investment decisions. C is incorrect. Return attribution, not risk attribution, attempts to identify investment management value added by decomposing the excess return into separate contributors to excess return from allocation and selection decisions relative to a benchmark.

8 B is correct. Return attribution should provide a complete understanding of a portfolio's excess returns.

A is incorrect because return attribution does not focus on measuring risk (risk attribution does). C is incorrect because return attribution compares the portfolio return to the benchmark return but does not compare the variability of those two returns.

9 B is correct. One of the attributes of an effective return attribution process is to reflect the investment decision process.

A is incorrect. Ranking portfolio managers is not part of an attribution process. C is incorrect because excess returns can be effectively calculated and attributed in either arithmetic or geometric terms.

10 C is correct. If the return generated by the return attribution does not reconcile to the return presented to the client the attribution will be considered incomplete and the quality of the analysis brought into question.

A is incorrect. Return attribution analysis can be used by portfolio managers to explain performance to clients. B is incorrect. Selection of an appropriate benchmark is made on an *ex ante* basis based upon the portfolio manager's intended investment strategy. It is not the role of return attribution to validate the benchmark.

11 B is correct. $(w_i - W_i)B_i$ represents the contribution to allocation for the ith sector in the Brinson–Hood–Beebower (BHB) model. In the Brinson–Fachler (BF) model, the contribution to allocation is $(w_i - W_i)(B_i - B)$. In the BHB model, B_i, or the return of the benchmark in the ith sector, could be negative and (given that $w_i > W_i$) would imply a negative allocation contribution. In the BF model, contribution to allocation would be negative only if the sector underperformed the overall benchmark, B.

A is incorrect because the formula for security selection are the same in the BF model and in the BHB model. C is incorrect because both models produce the same allocation effect at the portfolio level, $B_S - B$.

12 C is correct. The Brinson–Fachler (BF) model calculates the allocation effect as $(w_i - W_i)(B_i - B)$. In this case, the BF allocation effect in the consumer goods sector would be (0.6 – 0.5)(–0.04 – –0.05) = (0.10)(0.01) = 0.001 or 0.1%. The BF model takes in to account the fact that the manager was overweight a sector that outperformed the fund's benchmark, thus making a positive contribution to excess return.

A is incorrect as it is the Brinson–Hood–Beebower (BHB) allocation effect. The BHB allocation effect is given as $(w_i - W_i)B_i$. In this case, the BHB allocation effect in the consumer goods sector would be (0.6 – 0.5)(–0.04) = –0.004 or –0.4%. The BHB model does not take into account how the sector benchmark (B_i) performance relative to the overall benchmark return, B. B is incorrect because the use of the BF model implies a return attribution calculation, and thus a reference to a benchmark.

13 A is correct. An underweight position in sector i means that $(w_i - W_i)$ is negative. Given a negative sector benchmark return, B_i, the sector allocation effect $A_i = (w_i - W_i)B_i$ as the product of two negative numbers will always be positive.

14 B is correct. The only difference between the two models is the calculation of sector allocation effects, that is, $[(w_i - W_i) B_i]$ (BHB model) versus $[(w_i - W_i)(B_i - B)]$ (BF model).

A is incorrect. Both models use the same formula to calculate the interaction effects: $[(w_i - W_i)(R_i - B_i)]$. C is incorrect. Both models calculate selection effects as: $[W_i(R_i - B_i)]$.

15 A is correct. The selection effect of Energy is

$$W_i(R_i - B_i) = 25\% \times (7\% - 5\%) = 0.5\% > 0$$

B is incorrect because the allocation effect of Health Care is

$$(w_i - W_i)B_i = (25\% - 25\%) \times 9\% = 0$$

C is incorrect because the interaction effect of Information Technology is

$$(w_i - W_i)(R_i - B_i) = (25\% - 30\%) \times (3\% - 2\%) = -0.05\% < 0$$

16 B is correct. Using Equation 9, $S_i = W_i(R_i - B_i)$, we have:

Banking: 0.3(–0.02) = –0.006

Transportation: 0.45(0.03) = 0.0135

Communications: 0.25(–0.015) = –0.00375

Total: –0.006 + 0.0135 – 0.00375 = 0.0038, or 0.38%

A is incorrect. It is the sum of excess return but does not adjust for the benchmark weight in each sector. C is incorrect; it is the total benchmark return (–5.0%) multiplied by the total excess return (–0.5%).

17 A is correct. The formula to calculate the allocation effect for the *i*th segment is $A_i = (w_i - W_i)B_i = (0.5 - 0.7) \times 5.1\% = -1.02\%$ or –1.0%.

B is incorrect because 0.0 is the selection effect for the large cap segment. C is incorrect because 0.1% is the allocation effect using the Brinson–Fachler model: (0.5 – 0.7)(5.1% – 5.8%) = 0.14% or 0.1%

18 A is correct. The decision to underweight energy added 1.29% to the portfolio return but the security selection within energy was –0.10%. Ignoring interaction, energy was a positive contributor of 1.19% to overall performance.

B is incorrect. The attribution analysis shows a 0.30% in selection attribution; this indicates the manager outperformed the market and did add value in security selection but this was not the largest sector contributor to performance. C is incorrect. The contribution to overall performance from the allocation strategy of consumer staples was +1.40%; however the selection decisions cost the portfolio –0.25% resulting in a positive 1.15% in performance. This was not the largest sector contributor.

19 C is correct. Using Equation 9, $S_i = W_i(R_i - B_i)$, we have for the technology segment: 35%(22% – 19%) = 1.05% or 1.1%

B is incorrect. In this case the portfolio weight, rather than the benchmark weight, was used to calculate the selection attribution: 30%(22% – 19%) = 0.9%. A is incorrect. It is the interaction return: (30% – 35%) × (22% – 19%) = –0.15% or –0.2%.

20 B is correct. Using the Brinson–Fachler model, the allocation effect is measured by $(w_i - W_i)(B_i - B) = (0.60 - 0.50)(0.08 - 0.056) = 0.0024$ or 0.24%.

A is incorrect. It represents the contribution from selection: 0.50(0.07 – 0.08) = –0.005 or –0.50%. C is incorrect. It was found using the Brinson–Hood–Beebower model: (0.60 – 0.50)(0.08) = 0.008 or 0.80%.

21 C is correct. Interaction is a directly calculable effect resulting from the combination (or interaction) between allocation and selection effects.

A is incorrect because the interaction effect need not be small and typically will be greater for larger allocation bets. B is incorrect because interaction is not part of the investment decision making process, as portfolio managers do not seek to add value through interaction decisions.

22 A is correct. The formula for geometric excess return is $(1 + S^G)(1 + A^G) - 1 = (1 + R)/(1 + B) - 1$. The portfolio return ($R$), the benchmark return (B), and the geometric allocation effect for the portfolio (A^G) are all provided. Entering everything into the equation gives:

$$\left(1 + S^G\right)\left(1 + 0.0019\right) - 1 = \frac{\left(1 + 0.057\right)}{\left(1 + 0.047\right)} - 1$$

Solving for S^G gives $S^G = 0.0076$ or 0.76%. Alternatively, S^G can be calculated from Equation 24:

$$S^G = \frac{\left(1 + R\right)}{\left(1 + B_S\right)} - 1 = \frac{\left(1 + 0.057\right)}{\left(1 + 0.049\right)} - 1 = 0.0076$$

(Note that B_S = 10% × 4% + 30% × 3% + 60% × 6%, using Equation 3).

B is incorrect because selection is calculated simply as a subtraction of the allocation effect from the excess return of the portfolio:

Selection = (Portfolio Return – Benchmark Return) – Allocation effect

Selection = (0.057 – 0.047) – 0.0019

Selection = 0.0081 or 0.81%

C is incorrect because it equals the total attribution effect or excess return.

23 B is correct. The contribution of the ith sector to geometric selection can be calculated using Equation 23:

$$S_i^G = \left(w_i\right)\left(\frac{\left(1 + R_i\right)}{\left(1 + B_i\right)} - 1\right)\frac{\left(1 + B_i\right)}{\left(1 + B_S\right)}, \text{ where } B_S = \sum_{i=1}^{n} w_i B_i$$

$$= \left(0.1\right)\left(\frac{\left(1 + 0.05\right)}{\left(1 + 0.07\right)} - 1\right)\frac{\left(1 + 0.07\right)}{\left(1 + 0.052\right)} \text{ for health care sector}$$

$= -0.0019$ or -0.19%.

A is incorrect because it is the arithmetic selection effect including interaction:

= 10% × (5% − 7%)

= −0.0020 or −0.20%

C is incorrect because it represents the geometric allocation effect:

$$= \left(w_i - W_i\right)\left(\frac{\left(1 + B_i\right)}{\left(1 + B\right)} - 1\right)$$

$$= \left(10\% - 20\%\right)\left(\frac{1 + 0.07}{1 + 0.053} - 1\right)$$

$= -0.0016$ or -0.16%.

24 A is correct. The contribution of the *i*th sector to geometric selection is

$$S_i^G = (w_i)\left(\frac{1+R_i}{1+B_i}-1\right)\left(\frac{1+B_i}{1+B_S}\right) = (w_i)\left(\frac{R_i-B_i}{1+B_S}\right)$$

The second term of the equation is positive since R_i (4%) is greater than B_i (3%). Therefore, increasing w_i (30%) would have increased S_i^G.

B is incorrect. The portfolio's allocation effect is given by

$$A^G = \frac{1+B_S}{1+B}-1 = \frac{1+\sum_{i=1}^{i=n} w_i B_i}{1+B}-1$$
$$= \frac{1+(0.10\times 0.02+0.30\times 0.03+0.60\times 0.06)}{1+0.043}$$

So increasing the weight of the health care sector (10%), and therefore decreasing the weight of either the utilities sector (30%) or of the consumer goods sector (60%), would have the effect of decreasing the numerator in this last equation since the benchmark return of health care (2%) is lower than the benchmark returns of the two other asset classes (3% and 6%).

C is incorrect. The contribution of the *i*th sector to geometric allocation is given by

$$A_i^G = (w_i - W_i)\left(\frac{1+B_i}{1+B}-1\right)$$

Since for the consumer goods sector, B_i (6%), is greater than B (4.3%), the second factor in the above equation is positive, so that any decrease in w_i will bring a decrease in A_i^G.

25 C is correct. Transactions-based attribution uses both the holdings of the portfolio and the transactions (purchases and sales) that occurred during the evaluation period in the calculation. Because information on purchases and sales is included in the calculation, transactions-based attribution is considered to be the most accurate.

A and B are incorrect. Transactions are not included in holdings-based or returns-based attribution and these methods are therefore less accurate than transactions-based attribution.

26 A is correct. Transactions-based is the most difficult to type of attribution to implement. To obtain meaningful results, the underlying data inputs (purchases and sales) must be complete, accurate and reconciled from period to period and included in the calculation process.

B is incorrect. Using actual transaction data will not produce residuals like holdings-based transaction does. C is incorrect. With a transactions-based method, all the data is available so that the entire excess return can be quantified, explained, and reconciled with the actual return presented to the client.

27 A is correct. Holdings-based attribution analysis can be calculated over any time period.

B is incorrect because transaction-based attribution is the most difficult and time consuming to implement. C is incorrect. Holdings-based analysis requires information on portfolio positions. Returns-based attribution is most appropriate when the underlying portfolio holding information is unavailable.

28 C is correct. The strategic asset allocation of the fund is the responsibility of the fund sponsor. In fact this is the only decision the sponsor makes when he does not manage funds internally.

A is incorrect because security selection in each portfolio is the responsibility of the portfolio manager. B is incorrect because tactical deviations from the strategic asset allocation are the responsibility of the investment manager.

29 B is correct. The allocation effect at the sponsor level is equal to $(w_i - W_i)(B_i - B)$ where i refers to total LT bonds: (85% – 80%) × (3.25% –2.90%) = 0.018%.

A is incorrect because it is the selection plus interaction effect that was measured instead of the allocation effect: 80% (3.15% – 3.25%) + (85% – 80%) (3.15% – 3.25%) = –0.085%. C is incorrect because it evaluates the decision of the LT portfolio manager: (55% – 50%) × (4.00% – 2.90%) + (30% – 30%) × (2.00% – 2.90%) = 0.055%.

30 C is correct. The allocation effect at the LT portfolio manager is equal to $\Sigma(w_i - W_i)(B_i - B)$ where the summation is made over corporate bonds and government bonds: (55% – 50%) × (4.00% – 2.90%) + (30% – 30%) × (2.00% – 2.90%) = 0.055%

A is incorrect because it is the selection plus interaction effect that was measured instead of the allocation effect.

= 50% (3.50% – 4.00%) + (55% – 50%) (3.50% – 4.00%) + 30% (2.50% – 2.00%) + (30% – 30%) (2.50% – 2.00%)

= –0.125%

B is incorrect because it evaluates the decision at the sponsor level: (85% – 80%) × (3.25% – 2.90%) = 0.018%.

31 A is correct. A multi-level return attribution analysis depends on who is evaluated and one cannot directly compare the results of two analyses when these analyses are about different entities responsible for the investment decisions. For example, in this situation, if the sponsor is held responsible for the decision to invest 85%/15% in LT/ST, all allocation effects will be attributed to the sponsor.

B is incorrect because we cannot directly compare the allocation effect of a return attribution analysis at a sponsor level and the allocation effect of a return attribution analysis at a portfolio manager level. C is incorrect because when the allocation effect is analyzed at the sponsor level, there is no allocation analysis left at the segment level.

32 A is correct. Value factor return (–0.43%) times the portfolio's overweight position (0.19) equals a negative excess return impact of –0.00082.

B is incorrect. The portfolio was underweight in its exposure to small cap stocks. C is incorrect. The largest contributor to negative excess return was the impact of the small cap premium factor (–0.00294), not the impact of equity risk premium (–0.00201).

33 B is correct. The contribution column indicates the overall contribution to excess return of each factor. The contribution was 0.00139.

A is incorrect. Although the factor return is positive, the impact may be positive or negative, depending on whether the portfolio is under- or overweight the factor. C is incorrect. Although the portfolio is overweight the factor, the impact may be positive or negative, depending on whether the factor is positive or negative.

34 B is correct. The portfolio has high exposures to both the small-cap return premium factor (1.23) and the value return premium factor (1.17).

A is incorrect. The significant difference in sensitivities to both the small-cap return premium factor (0.25) and the value return premium factor (0.53) suggests the manager is not attempting to track the benchmark closely. C is incorrect. Positive high sensitivity numbers (greater than one) suggest that the portfolio tends to hold/favor small cap and value-oriented equities.

35 A is correct. Factor models are more flexible than asset-grouping models in that they can be tailored very closely to the investment process.

B is incorrect. Asset-grouping models are often easier to understand in terms of investment decisions. C is incorrect. Factor models are generally applied to single asset class portfolios.

36 B is correct. There is a large variety of factor models used in the industry.

A is incorrect. Factor models are dependent on the specific factors used in the analysis. C is incorrect. Factor models can be expensive to use. If the factor model is particularly complex, data and computer software costs can be significant.

37 B is correct. If only one factor (e.g., such as yield-curve positioning) is considered to be important in evaluating an investment manager, then a Brinson attribution approach would be an appropriate model to follow. The allocation effect would account for the manager's decisions regarding yield-curve positioning with any other decisions included in selection and interaction.

A is incorrect. The allocation effect measured by the Brinson approach would account only for one of the factors supporting the investment decision process with all other decisions included in selection and interaction. It would be more appropriate to develop a specialized, multi-factor approach that can be more closely tailored to the actual investment decision process. C is incorrect. The allocation effect measured by the Brinson approach would account only for one of the three factors supporting the investment decision process with the other two factors included globally in selection and interaction.

38 C is correct. The Brinson approach captures one factor in the allocation effect and combines all other factors in the selection and interaction effects. Thus, it is imprecise in dealing with multi-factor models.

A is incorrect. Fixed income indexes that can serve as benchmarks are readily available. As long as the investment decision process can be segmented in the fixed income categories of the benchmark, the Brinson approach can be appropriate. B is incorrect. Exhibit 26 is an example of the Brinson approach analyzing the impact of decisions across the yield curve.

39 C is correct. The impact of interest rate management is 0.41% (= 0.08% + 0.02% + 0.31%).

A is incorrect. This calculation considers changes along the yield curve (i.e., 0.08%) as the only component of interest rate management. B is incorrect. This calculation considers duration and convexity (i.e., 0.31% + 0.02%) as the only components of interest rate management.

40 C is correct. The manager's selection skill contributed the most (–0.26%) of any factor to the portfolio's underperformance.

A is incorrect. The manager's contribution to excess return due to interest rate management was 0.15% (= –0.02% + 0.03% + 0.14%). B is incorrect. The manager's skill at interest rate management exceeded his sector allocation ability (0.15% versus 0.07%).

41 A is correct. All else equal, (i) using shorter time periods in measuring holdings will yield more accurate results than longer time periods (this makes A better than C); and (ii) a portfolio with low turnover will likely have fewer transactions

(transactions cannot be not fully accounted for by holdings-based attribution and they lead to residuals) than a high turnover portfolio (this makes A better than B and A better than C).

B is incorrect because a portfolio with high turnover will lead to less accurate analysis in holdings-based attribution. C is incorrect because the use of longer time periods and a portfolio with high turnover will lead to less accurate analysis in holdings-based attribution.

42 C is correct. Holdings-based attribution suffers from greater residuals than transaction based attribution, due to the incomplete treatment of transactions.

A is incorrect because arithmetic excess returns do not add up over multiple periods and this will lead to a residual unless a geometric attribution methodology is used. Thus, switching to geometric attribution model will decrease residuals. B is incorrect because improving the quality of the data inputs so that the segment level and total level data are consistent, will increase the accuracy of the attribution results and decrease the residuals.

43 C is correct. By using beginning of period holdings and ignoring transactions (as in performing holdings-based attribution analysis), a residual effect called the timing or trading effect is produced.

A is incorrect because the choice of the benchmark itself, although it may affect the quality of the return attribution analysis, does not cause a residual effect. B is incorrect because a residual is more likely to occur in multi-period analysis than single period analysis, and more so with arithmetic return attribution.

44 B is correct. The selection (including interaction) effect of information technology is 10% × (3% – 5.5%) = –0.25%.

A is incorrect because it is the allocation effect assuming a sector allocation decision: (10% – 0%) × (2% – 5.5%) = –0.35%. C is incorrect because it is the selection effect assuming a sector allocation decision: 10% × (3% – 2%) = 0.1%.

45 C is correct. The return attribution of off-benchmark decisions depends on their rationale. This is clearly a sector allocation decision and should be measured as such. The calculation is (5% – 0%) × (7.10% – 2.05%) = 0.25%.

A is incorrect. This was calculated as (5% – 0%) × (0.00% – 2.05%) = –0.10%. B is incorrect. This was calculated as a selection effect: 5% × (10.80% – 7.10%) = 0.185% or 0.19%.

46 B is correct. The return attribution of off-benchmark decisions depends on their rationale. This is clearly a selection decision. The benchmark chosen for the consumer staples sector should be the overall benchmark return which will lead to a zero allocation effect.

A is incorrect. This is a selection decision, not a sector allocation decision. The selection effect would be zero only if it had been an allocation decision and if it had been impossible to identify an appropriate sector index. C is incorrect. Only the allocation effect is zero.