Validity of NPV rule and IRR criterion for Capital Budgeting and CBA

Abstract

The validity of the NPV rule and the criticisms against IRR are evaluated using a capital amortization approach (CA) and the results presented. Negative or zero or no IRR is consistent with the net cashflow (NCF) when the sum of NCF is either zero or negative. Reinvestment, with non-normal NCF, causes spurious NPV and multiple IRR. These problems are resolved by eliminating reinvestment. With normal NCF, reinvestment affects NPV, in some cases, but not the IRR. Finally, IRR can rank mutually exclusive projects and can be estimated either as a return on total or balance of capital.

The NPV can be used with normal NCF investment and the result will be consistent with the IRR. The question is whether the investors are interested to base their decision on the return as a percentage (IRR) or as an absolute money value (NPV) or as per dollar of investment (BCR or PI). The caution is that the even with normal NCF the NPV does involve reinvestment (at low discount rates) but not the IRR. Also, while ranking mutually exclusive investments with normal NCFs, the ranking by NPV changes with discount rates but ranking by IRR is consistently unique. With non-normal NCFs the NPV is spurious and IRR not unique. The CA approach discussed in this paper resolves this problem by eliminating reinvestment. The NPV rule, therefore, needs revisiting.

JEL classifications: C60, C63, D61, E22, E40, G3, G24, G31, H43, O2, O12, O16, O2

1. Introduction:

The debate on the preferred criterion (IRR vs NPV) for capital budgeting continues but the 'NPV rule[1]' dominates the debate among most academics and some managers. The proponents of NPV rule argue that the problem of multiple or negative or no IRR universally regarded as a fatal flaw for the IRR method (see Hazen, 2003; Kierullf 2011; Osborne 2010; Brigham et al. 2012). According to Merlo (2012), with non-monotonic[2] NPV function originating from non-normal NCF (NNCF), using the NPV rule, as if NPV is without errors, and to abandon the IRR is incorrect. In such cases, the NPV increases with an increasing discount rate that could lead to a wrong investment decision. In other words, no project would become more profitable if the cost of capital was higher. Merlo (ibid) concluded that 'it is wrong to treat symptoms of disease when in fact it is necessary to diagnose its causes'. Merlo presented some important conceptual problems that are carefully evaluated and addressed,

With normal NCF, the IRR criterion will give the same answer as the NPV. Theoretically, both NPV and IRR are estimated by solving the same NPV function for a desired hurdle rate (cost of capital (CoC) or weighted average cost of capital (WACC)) and for an unknown rate, respectively. That being the case, if there are problems with IRR, then the NPV must have been equally affected. Applying the NPV rule, without further investigation of the real problem is a sort of bounded rationality[3] approach. Such a bounded rationality model might lead to suboptimal decision-making. Such an on-going serious criticisms would continue to undermine the value of CBA and its application in capital budgeting, valuation and in regulatory analysis. Under that context, the

[1] The NPV rule is to invest in projects that have a highest positive net present value (NPV).
[2] A NPV function is non-monotonic when it increases as wells as decreases at intervals with increasing discount rates.
[3] For definition see Simon 1982.

present analysis is important to evaluate the robustness of the NPV rule and the validity of controversies surrounding the CBA criteria (IRR or NPV or BCR) and to inform the analysts and decision makers appropriately.

Other commonly prevailing arguments to justify NPV rule includes are that the IRR is not suitable for ranking and selecting mutually exclusive projects or projects with multiple IRR (for e.g. Osborne 2010, Kierulff 2012, World Bank 2013, Asian Development Bank (ADB) 2017, Juhász 2011). Analysts and decision makers, therefore, opt for NPV as the better criterion to accept or reject projects or rank mutually exclusive projects. Many texts also discuss that IRR involves reinvestment at a rate equivalent to IRR whereas NPV only reinvest at cost of capital. Also, critics of IRR argues that the IRR is not the return on the total capital but on the balance of capital (see Kelleher and MacCormack, 2004). The problems with IRR are real challenges but then the superiority of NPV rule must be proved beyond any doubts. The analyses and the findings presented in this paper are expected to contribute substantially to verify the validity of some of these arguments and to resolve other criticisms.

The purpose of this paper is to:

a. provide a better analytical insight into the problems of negative, zero or no IRR;
b. evaluate the cases of reinvestment of intermediate income and the impacts on IRR and NPV;
c. introduce new approaches to resolve the problems of multiple IRR and spurious NPV;
d. assess the suitability of IRR to rank mutually exclusive investments; an
e. estimate IRR as a return on total and balance of capital.

2. Literature Review:

Currently, IRR and NPV are being estimated using the net cash flow (NCF) data and the discounted cash flow (DCF) method. Arjunan (2019) discussed about a capital amortization (CA) approach to CBA and shown that the CA approach is more transparent and enables resolution of most of the problems associated with IRR and NPV. When the sum of undiscounted NCF is either negative or zero the investment ends up with a loss. With such cashflow, the estimated IRR may be zero or negative or multiple (see also Ben-Horin and Kroll 2012; Arjunan 2019). Ross (1995) did a detailed analysis and concluded that 'for most investments, the usefulness of NPV rule is severely limited' where an investment does not preclude alternative investment options.

Merlo (2016) discussed the consequences of the non-monotonic of the NPV function related to NNCF investments and concluded that the popular DCF method is not appropriate to estimate NNCF and the estimated IRR and NPV are not correct. Weber (2017) introduced a selective IRR (SIRR) criterion that he claimed to be equivalent to the NPV-rule. He argued that "an investor with a cost of capital of r = 10% would report the return of a project with the cash-flow stream (-5, 16, -12) as minus infinity (and therefore completely unacceptable), whereas an investor with a cost of capital of r = 25% would report the return of the same cash-flow stream as 100% which is very attractive indeed." As could be seen, the problem is when the cumulative undiscounted sum of the above NCF is -1 (cumulative loss). That being the case, a negative or zero IRR is possible. A 100% return is highly unrealistic and purely a symptom associated with NNCF that is not supported by the project benefit stream (sum of NCF = -1, i.e. net loss). In litigations involving CBA in the USA and Australia, there are judgements that question the validity of the analysis (for example Conn, 2013; Albany & Ors v The Commonwealth of Australia 1976).

Ben-Horin and Kroll (ibid) concluded that there are two reasons for the preference of NPV ranking over the IRR ranking: ‘1. The NPV is an absolute measure of wealth, whereas IRR is a relative measure of wealth, and 2. The time value of money employed in calculating the NPV is the risk-adjusted cost of capital, which is a measure of the actual economic opportunity cost of the capital invested in the project. On the other hand, the time value of money employed in calculating the IRR is the IRR itself, which is an artifact of the project’s cash flow and does not represent an economic alternative cost.’ These reasons are invalid for three reasons: a. NPV and IRR are estimates produced by the same DCF methods; b. they use the same NCF data and applying the time value of money; and c. both can be compared with the risk-adjusted cost of capital. When the IRR is above the risk-adjusted cost of capital the project is accepted.

World Bank (2013) and ADB (2017), in their Guidelines on Investment Project Analysis, indicated that the IRR is not suitable for the ranking of competing projects. Weber (2014) reported that the IRR is generally considered inferior to the net present value (NPV) as a tool for evaluating and ranking projects. Cuthbert and Cuthbert (2012) argued that IRR is potentially misleading. Some others concluded that IRR is a limited decision tool and advised the financial analysts to use it with caution (see also Kierulff 2012 and Kelleher 2004). Phalippou (2009) stated that IRR has its own shortcomings and biases and creates room for managers to manipulate the performance reporting. Park (2005) recommended to abandon the IRR analysis and use the NPV criterion whenever analysts encounter multiple rates of return.

Some authors reported that IRR and NPV together guarantee the making of relevant decisions (for example Juhász, 2011, Hall and Millard, 2010). Jacobs (2007) argued that ‘the conflict between NPV and IRR arises because of misinterpretations that have been made. The NPV-method and the

IRR-method are not two measures of investment worth - as it is reported in many textbooks - but just one single method'. His argument is right in the sense that the same NPV function is used to estimate NPV and IRR by DCF method. NPV is estimated by solving the function for a known discount rate whereas IRR for an unknown discount rate that makes the NPV = 0. Both NPV and IRR are mathematically interrelated (IRR is the discount rate that makes NPV Zero and BCR 1).

Berkovitch and Israel (2004) showed that the use of NPV as an investment criterion leads to inefficient capital budgeting outcomes and therefore other capital budgeting criteria, like the IRR and the profitability index (PI) will continue to dominate. Hazen, 2003, reported that the problem of multiple or nonexistent IRR is not really a flaw at all, and can be easily dealt with conceptually and procedurally. Several authors are of the view that multiple internal rates constitute a severe drawback and not helpful (Brealey and Myers 1996, Canada et al. 1996, Sullivan et al. 2000, White et al. 1998, Eschenbach 1995 and Park 1997). Arjunan (2019) studied the properties of non-monotonic NPV function associated with NNCF investments and concluded that such NPV function leads to spurious NPV and multiple IRR. Arjunan (ibid) also presented a CA approach to resolve these problems.

Some authors argue that these conflicts are caused by the problems of scale of investment, timing of cashflow, different maturity or life of investment (for example Carter et al 1997 and Barney and Danielson 2004). These are not problems but are associated with the nature of the investment and cashflow. Given that timing of NCF, the most profitable investment must be identified. Barney and Danielson (2004) compared mutually exclusive projects and concluded that differences in return duration cause ranking conflicts between NPV and IRR. They proposed to consider duration or generalized NPV before making investment decisions when faced with such ranking conflicts.

According to Mauboussin and Callahan (2014) return on invested capital (ROIC) is one measure of a company's capital efficiency and that the ROIC analysis can provide insight into the sources of a company's competitive advantage. Damodaran (2007) was of the view that growth unaccompanied by excess returns (ROIC) creates no value. The ROIC analysis is therefore important both for selection of investment and for ranking mutually exclusive investment

In summary, both IRR and NPV are criteria estimated by the same DCF method with time value of money concept, using the same NCF data and both estimates are mathematically interrelated. From an analytical or mathematical perspective, NPV is a point estimate (at hurdle rate) whereas the IRR estimate involves evaluating a range or profile of NPVs at various discount rates to locate the discount rate that makes the NPV = 0. Instead of a point estimate (NPV), if a NPV profile review is undertaken, the adequacy or inadequacy of NPV could be better explained and appreciated. This paper investigates and presents numerical evidence to better appreciate the capital investment criteria, NPV and IRR.

3. Methodology

Some of the key issues relating to CBA are discussed that would facilitate a better appreciation of the methodology. Any investment decision involves estimation of two components of return: 1. return of capital (ROC) or recovery of capital invested; and 2. a desired return on invested capital (ROIC). The ROIC is measure of the return earned on capital invested. The DCF method makes use of the NCF data to estimate the ROC and ROIC.

Mathematically, IRR is the discount rate that makes the NPV = 0 and the BCR = 1. These relationships among IRR, NPV and BCR remain consistent for normal and non-normal NCF. Normal positive cash flow stream (normal or orthodox NCF) refers to non-negative net cash flow all through other than the investment year (Year 0). In the case of the non-normal NCF, the cash flow of a project changes signs more than once, e.g. if one or two negative flows are followed by some inflows. In these cases, the project may have more than one IRR and the NPV will be zero as many times as there are multiple IRR. Now, some of these controversies or criticisms relating the IRR and NPV as decision criteria are evaluated using some of the methodologies discussed below.

Estimation of IRR and NPV: Equation 1 is commonly being used to estimate the IRR and NPV.

$$NPV = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + ...\frac{CF_t}{(1+r)^t} - CF_0 \qquad \text{Equation 1.}$$

Where, CF_1, CF_2… CF_t are the net cash inflow during periods 1 to t; CF_0 is the capital invested; ‘r’ is the discount rate. NPV is estimated with hurdle rate or cost of capital as the discount factor ‘r’. IRR is identified by using a range of discount rates (r). The ‘r’ that makes the NPV = 0, is the IRR. While the IRR remains constant for a NCF, the NPV keeps changing for the same NCF with changes in the discount rate ‘r’ and the NPV is sensitive to discount rates. In the Eq.1, the sum of the right-hand side (RHS) variables (other than the CF_0) is the discounted net cash inflow or the PV of net cash inflows (PVcf). PVcf minus CF_0 is the NPV as per Eq.2 that is derived by substituting PVcf on the RHS in Eq.1.

$$NPV = PV_{cf} - CF_0 \qquad \text{Equation 2}$$

This equation is useful to interpret the relationship between NPV and IRR. First, as per Eq.2, NPV is the unutilized PVcf after recovery of the capital (CF_0 or ROC) and recovery of ROIC at hurdle rate. The PVcf is also the remaining ROIC in absolute term after the recovery of ROC and a part of ROIC in percentage terms (hurdle rate). The PVcf keeps changing for every change in the discount rate (r). The discount rate used represents the ROIC. ROIC is the earning on the unrecovered balance of an investment. As such, the discount rate (ROIC) used in the denominators of each present value (PV) computation is critical in determining what the final NPV number will turn out to be. A small increase or decrease in the expected or desired ROIC (r) will have a considerable effect on the final output of NPV. Several interpretations can be made from the Eq.2. When r = IRR, the NPV will be '0' that indicates the full utilization of the PVcf to pay-off the CF_0 (ROC) and the highest possible ROIC (= IRR). With IRR as the ROIC, the investment income is optimized. When r < IRR, the NPV is positive. The positive NPV represents the unutilized PVcf (see Eq.2). The lower discount rate (r less than IRR) is inadequate to fully utilize the PVcf and to maximize the ROIC (ROIC < IRR). When r > IRR, the NPV is negative. Here, the PVcf is not adequate to support that higher ROIC (r higher than IRR) and therefore the negative NPV. The analysis is further expanded with the capital amortization analysis.

Capital amortization schedules (CAS): Capital amortization schedule is a table or chart showing how much of each periodic future income or return from an investment is going towards interest payments or return (ROIC) and capital (principal) recovery (ROC). CAS virtually accounts for the ROC and ROIC before discounting. CAS analysis will indicate when the net cash inflow (NCF) is fully utilized (closing balance zero) to recover the ROC and a ROIC that remains invested. Closing balance of >0 and <0 reveals that there is excess (unutilized) benefit at that ROIC or benefits not sufficient to support the required ROIC, respectively. The PV of the final closing balance is the NPV (see Arjunan 2019) that will be 0 when the ROIC is equal to IRR or >0 when the ROIC < IRR (underestimate) or <0 when the required ROIC > IRR (unachievable). CAS estimation follows equation 3.

$$\sum_{t=1}^{n} (OB_t * (1+R) - NCF_t) = CB_t \qquad \text{Equation 3}$$

Where OB is the opening balance of capital each year, ‘r’ is the interest rate (or return r), NCF is the net cash inflow or the intermediate net income per year, CB is the closing balance each year and ‘t’ is year 1 to n (n is terminal or final year of project life or maturity of investment). The CB of a year is the OB of the next year. The CAS and DCF equations (eq. 2 and 3) are merged that shows the NPV as the PV of the final closing balance and the rate that makes the final closing balance zero is the IRR in CAS:

$$PVCB = \frac{CF_1}{(1+r)^1} - CF_0 = NPV \qquad \text{Equation 4}$$

An illustrative CAS with reinvestment is presented in Table 1 to discuss the CA methodology. The PV of CB -171 is the NPV of -128 at the hurdle rate or cost of capital of 10% (see last row Table 1). IRR is the interest rate that makes the final CB zero. IRR can be estimated simply by using the “goal seek function” in excel. Goal seek function can iteratively estimate the IRR by changing the interest rate until the CB = 0. In this case, the CB will be zero at discount rates of 0%, 100% and 200% (not displayed in Table 1) and therefore there are three IRRs (multiple IRR).

Table 1: An Illustrative CAS with Reinvestment and Multiple IRR

Years	Note	0	1	2	3	IRR	NPV	
Cash flow*		-1000	6000	-11000	6000	**0.0%**	At 10% by DCF	-128
Opening Balance	OB		-1000[a]	4900[c]	-5610	100%		
Interest/ Return – R	10.0%		-100	**490[d]**	-561	200%	At 10% by DCF Without reinvestment	
Income flow	I		6000	-11000	6000			-533
Closing Balance	CB		4900[b]	-5610	-171		At 10% by CAS	-128

- Data Source: http://www.exceluser.com/formulas/irr.htm

1. a. Capital investment; b. CB = OB+R -I; c. OB = CB of previous year; d. Reinvestment (positive)
2. PV of final CB (-171) is the NPV at that rate (-128) and discount rates ‘0’, 100% and 200% (Multiple IRRs) make the final CB ‘0’.

The above CAS reveals, multiple IRR of 0%, 100% and 200% and the DCF method also the gives the same results. The CAS method is a perfect substitute for DCF method but is more transparent.

In year 2, the interest is positive that means an income flow that is reinvestment income from the positive OB of \$4900 in year 2. As the sum of NCF is ‘0’ (no return), the IRRs 100% and 200% are spurious caused by reinvestment of \$490. Similarly, the NPV without elimination of the reinvestment is -128 which is also spurious. Without reinvestment the IRR = 0 and the NPV at 10% will be -533. IRR ‘0’ is consistent with NCF whose sum without discounting is ‘0’. This methodology is applied throughout the analysis.

The CAS model enables identification of any reinvestment of the intermediate income. The interest or return row for all the three years are expected with negative signs (interest expenses or return paid out to investors). In the case of year two, the interest rows have positive signs that will be an interest income i.e. reinvestment income. Such a situation eventuates because of the positive OBs in year two. In cases, where there are positive OBs in some years there will be naturally positive interest income that is reinvestment. In those cases, the CAS method is modified (MCAS) to exclude interest income flow i.e. reinvestment from the investment income flow (interest on positive OBs). The “if function” in excel is used to eliminate positive interest income. The impact of excluding reinvestment on IRR and NPV can be evaluated.

A simplified method to estimate Benefit Cost Ratio (BCR) of profitability index (PI) (Eq 5) is also used. BCR or PI is the ratio of present value (PV) of benefit to the PV of cost. The CB is the excess of income after recovery of CF_0. Therefore, CF_0 + PV of CB = PV of benefit; The PV of the CB is the NPV. CF_0 is the PV of capital cost. BCR is estimated using the formula, $(CF_0+NPV)/CF_O$ and the reduced form of this equation is:

$$BCR = 1 + \left(\frac{NPV}{CF_0}\right) \quad \text{Equation 5}$$

This simplified new formula can be used to estimate BCR both under the CAS and DCF methods. In the next section, the results are presented and discussed.

4. Results and discussions:

The discussions in this section is organized into four sub-headings: a. No IRR or negative IRR or zero IRR; b. reinvestment of intermediate income and resolution to multiple IRR; c. mutually exclusive investment cases; and d. return on total capital vs return on remaining capital.

4.1 No IRR, Negative or Zero IRR

Conceptually, if there are multiple IRRs, then the NPVs will also be zeros as many times as the number of IRRs. The problem of multiple IRR and corresponding multiple zero NPVs are caused by the NNCF data. Similarly, the problem of 'no or negative or zero' IRRs is caused by the some of the NNCF (rarely NCF too) investments. The discussions in this section starts with the cases of "no or negative or zero IRR". The results of numerical analysis conducted are presented in table 2 and summarized here:

- NCF 1 (see table 2) does not have any capital investments and the NCF for investment year (zero year) is zero. There is no need for the estimation of IRR or to recover capital invested (ROC) when there is no capital invested (i.e. no negative inflow[4]). Naturally, there will be no IRR Such NCF (with all positive cash flows) naturally leads to reinvestment of the intermediate income ($846).

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[4]. According to "Descartes' rule of signs" there can be as many different IRRs as there are changes in the sign of the flows ("- + -" is two sign changes).

- With NCF 1 data (with all positive cash flows), the estimated NPV by CAS[5] or DCF method, that includes reinvestment income ($846), is $5359. For the same data, the estimated NPV by MCAS method, that does not include reinvestment income, is $4781. The NPV is also misleading because the estimate includes reinvestment at the hurdle rate (cost of capital CoC). In the absence of capital investment, the NPV equation collapses to PV equation and the estimated NPV = PV of benefits. In such a situation the analysis is not capital budgeting but estimation of the present value of the benefits.

Table: 2. NCF and NNCFS - Estimated IRR and NPV by DCF/CAS and MCAS Methods

Details	NCF 1	NCF 2	NCF 3	NCF 4	NCF 5	NCF 6
Year 0	0	-1000	-1000	-10000	-10000	-50000
Year 1	1000	900	900	5000	5000	50000
Year 2	1500	-300	-300	-7000	-6000	30000
Year 3	2000	900	1000	7000	7000	-40000
Year 4	2500	-600	-600	4000	8000	30000
Sum of NCF (SNCF)	**7000**	**-100**	**0**	**-1000**	**4000**	**300**
IRR by CAS/DCF[6]	No IRR	No IRR	0.0%	-3.3%	11.1%	26.9%
NPV at 10% (CAS/DCF)	$5,359	-$163	-$88	-$3248	$310	$10,686
Reinvestment	846	900	600	0	0	2450
IRR by MCAS Method	No IRR	-7.4%	0.0%	-3.3%	11.1%	23.3%
NPV at 10% (MCAS)	$4781	-$186	-$118	-$3248	$310	$8845

- The NCF 2 is a NNCF and the sum of NNCF is negative (-$100). With such NNCF data, there is no IRR under CAS or DCF method. A CAS analysis, that includes reinvestment income of $900, reveals that at zero interest rate the CB is -$100, the NPV is -$100 at 0% and -$163 at 10%. A MCAS analysis, that excludes reinvestment, indicates that the CB is zero at -7.4%,

[5]. Due to space constraints, the results of CAS and MCAS analyses are furnished here but the analyses are available with the author.
[6]. Estimates by CAS and DCF perfectly match (see Arjunan, 2017).

that means the IRR is -7.4%, and NPV at 10% is -$186. With such NNCF investments, neither the IRR nor NPV is reliable because they are distorted by the reinvestment income.

- The un-discounted NCF is ‘0’ in the case of NCF 3 (a NNCF). The IRR is ‘0%’ for NCF3 and the cash inflow completely off-sets the capital and operating costs before discounting. Such situations are quite normal and that does not mean IRR is an unsuitable criterion. There is difference between the NPVs at 10% under CAS/DCF (-$88) and MCAS methods (NPV is -$118) that is caused by the reinvestment income. The NPV is therefore spurious and cannot provide a better investment criterion in these cases. The IRR is consistently zero and truly reflects that the investment could not generate return (ROIC).
- NCF 4 is again a NNCF and the sum of undiscounted NNCF is -$1000. The CAS or MCAS prepared at IRR of 0% leaves a CB of -$1000 under each method. However, at IRR of -3.3%, the CB is ‘0’ but the ROIC is unrecovered and therefore the IRR is consistently negative (-3.3%). There is no reinvestment and therefore no difference between IRR or NPVs at 10% estimated under both CAS / DCF and MCAS methods. Negative IRR is again not a problem, but a true reflection of the intrinsic worth of the NNCF.
- NCF 5 and 6 are also NNCFs with sum of their undiscounted NNCF positive. NCF 5 does not have reinvestment income as there is no positive OB under the CAS. The estimated IRR and NPV are consistent under CAS and MCAS method. However, with NCF 6, there is positive OB in year three that adds reinvestment income ($2450). The reinvestment income distorts both NPV ($10686 and $8845) and IRR (26.9% and 23.3%) under CAS/DCF and MCAS method.

In summary, these findings illustrate that cases of “no or zero or negative IRR” are the true reflection of the intrinsic value of the NCF and are consistent with the NNCF investments. Such

NNCF, with reinvestments, leads to spurious NPV as well as IRR. The widely accepted “NPV rule” needs re-examination, given the fact that NPV does suffers and spurious with NNCF. The MCAS method resolves these problems and further analyses are discussed in the next sections.

4.2 Reinvestment of intermediate income and Multiple IRR:

The most common assertion is that the IRR involves reinvestment at IRR and NPV at hurdle rate. The validity of this assertion or assumption is investigated using both NCF and NNCF investments. IRR and NPV at 10% (hurdle rate) are estimated both by CAS (the results are similar to DCF estimates). The CAS method transparently reveals reinvestments, if any. The MCAS method estimates IRR and NPV for those NCF and NNCF investments after elimination of the reinvestment. The results are furnished in table 3.

a. With normal NCF (investment A), the opening balance (OB) in all years (1 to 4) is negative and therefore there is only interest expenses and no interest income or no reinvestment income is included. As there is no reinvestment, there is no difference between the IRRs (18.3%) and NPVs ($183.9) estimated under CAS and MCAS methods. There is neither reinvestment at IRR nor at hurdle rate (10%) in the case of NPV (see Table 3).

b. Another normal NCF (Investment B) is evaluated. The estimated IRR does not involve reinvestment but NPV of $403 at 10% does involve reinvestment in years 3 and 4 (in bold numbers). When the MCAS method (that eliminates reinvestment; see years 3 and 4) is used the NPV at 10% is $376. **This is an important finding that the with normal NCF the reinvestment does affect NPV in some cases but not the IRR. In such a case, the NPV is spurious and the NPV rule is misleading.**

Table: 3. An Analysis of Reinvestment of Intermediate Income at IRR or Hurdle Rates

Year	0	1	2	3	4
A. NCF Investment - CAS at IRR: 18.03% - No reinvestment					
NCF	-1000	300	400	500	300
Opening Capital		-1000	-880	-639	-254
Interest at IRR: 18.03%		-180	-159	-115	-46
Income		300	400	500	300
Closing Balance		-880	-639	-254	0
A. NCF Investment - CAS at 10% NPV = $184 (PV of 269) - No reinvestment					
Opening Capital		-1000	-800	-480	-28
Interest at Hurdle Rate: 10%	0	-100	-80	-48	-3
Income		300	400	500	300
Closing Balance		-800	-480	-28	269
B. NCF Investment - CAS at IRR: 31.7% - No reinvestment					
NCF	-1000	600	600	300	200
Opening Capital		-1000.0	-716.5	-343.3	-151.9
Interest at IRR: 18.03%	31.7%	-316.5	-226.8	-108.6	-48.1
Income		600.0	600.0	300.0	200.0
Closing Balance		-716.5	-343.3	-151.9	0.0
B. NCF Investment - CAS NPV at 10% : $403 - With Reinvestment					
Opening Capital		-1000	-500	**50**	**355**
Interest at IRR: 18.03%	10%	-100	-50	**5**	**36**
Income		600	600	300	200
Closing Balance		-500	50	355	591
B. NCF Investment - MCAS - NPV at 10%: $ 376					
Opening Capital		-1000	-500	50	350
Interest at Hurdle Rate: 10%	10%	-100	-50	0	0
Income		600	600	300	200
Closing Balance		-500	50	350	550
C. NNCF Investment - CAS at IRR: 26.9%					
NNCF	-50000	50000	30000	-40000	30000
Opening Capital		-50000	-13475	12894	-23631
Interest at IRR: 26.9%		-13475	-3631	3475	-6369
Income		50000	30000	-40000	30000
Closing Balance		-13475	12894	-23631	0
C. NNCF Investment - CAS AT 10%: PV of the CB: $15645 = NPV at 10%: $10685					
Opening Capital		-50000	-5000	24500	-13050
Interest at Hurdle rate: 10%		-5000	-500	2450	-1305
Income		50000	30000	-40000	30000
Closing Balance		-5000	24500	-13050	15645
B. NNCF Investment - MCAS at IRR: 23.26%					
Opening Capital		-50000	-11632	15662	-24338
Interest at IRR: 23.26%		-11632	-2706	0	-5662
Income		50000	30000	-40000	30000
Closing Balance		-11632	15662	-24338	0
B. NNCF Investment - MCAS at 10%: PV of CB: $12950 = NPV at 10%: $8850.0					
Opening Capital		-50000	-5000	24500	-15500
Interest at Hurdle Rate: 10%		-5000	-500	0	-1550
Income		50000	30000	-40000	30000
Closing Balance		-5000	24500	-15500	12950

c. Under the NNCF (investment C), the OB is positive in year 3 that leads to reinvestment income (interest income or positive interest) included under the estimate. With inclusion of reinvestment income, the estimated IRR and NPV for the NNCF (B) are 26.9% and $10685, respectively. Contrarily, the estimated IRR and NPV for the same NNCF (B) under the MCAS method (that excludes reinvestment income) are 23.26% and $8850. This illustrates that with some NNCF investment not only the IRR but also the NPV is an overestimate as they are equally affected by the reinvestment income. IRR and NPV estimated by MCAS method, that excludes reinvestment, are the appropriate estimates.

d. The assumption of reinvestment at IRR or at hurdle rate in NPV are false in the cases of normal NCF and with some of the NNCFs. However, such reinvestment is evident only with NNCFs that affects both IRR and NPV (but not at IRR or hurdle rate).

In summary: a. with normal NCF investments, there is no reinvestment of intermediate income, neither at IRR nor at hurdle rate in the case of NPV; b. Another normal NCF investment analysis reveals that reinvestment does affect the NPV and not the IRR; c. With NNCF investments, there is reinvestment with some (not all[7]) of the NNCFs (see also Arjunan 2017); c. the reinvestment is a problem associated with some NNCF[3] data and nothing to do with IRR or NPV, as assumed or asserted in most of the published works; and d. any modification in the estimation of IRR or NPV, based on reinvestment assumption without evidence is not appropriate. The resolution of multiple IRR is discussed next.

[7] CAS of a NNCF investment that has positive OB in one or more years, only will lead to reinvestment.

Multiple IRR: As discussed, multiple IRR is mostly associated with NNCF particularly if there is reinvestment of intermediate income. Some NNCFs without reinvestment income will not lead to multiple IRR. Projects A (Damodaran, 2010) and B (Chen 2008) with NNCF data, available in public domain, are selected for this analysis. In both projects, the NNCF is negative at year '0' and again at year 4 in both projects. These data are used to estimate the NPVs and IRRs and the results are presented in Table 4 and summarized here:

Table: 4. NNCFs and Estimated IRRs and NPVs by CAS/DCF and MCAS Methods

Year	NCF A	Estimates	By CAS/DCF	By MCAS
0	-1000	IRR (1)	6.6%	-9.0%
1	800	IRR (2)	36.5%	NA
2	1000	NPV at 6.6%	0	-$142.2
3	1300	NPV at 36.5%	0	-$193.4
4	-2200	NPV at 10%	$27.8	-$157.1
Sum of NCF (SNCF)	-100	NPV at 20%	$52.5	-$183.3
		NPV at -9%	-$396.3	0
Year	NCF B	Estimates	By CAS/DCF	By MCAS
0	-580	IRR (1)	9.9%	-12.1%
1	530	IRR (2)	32.2%	NA
2	530	NPV at 9.9%	0	-$94.3
3	530	NPV at 32.2%	0	-$109.0
4	-1080	NPV at 10%	$0.4	-$94.8
Sum of NCF (SNCF)	-70	NPV at 20%	$15.6	-$105.7
		NPV at -12.1%	$-319.8	0

The sum of undiscounted NCF (SNCF) for projects A and B are -$100 million and -$70 million, respectively. As there is net loss or the net income is negative even without discounting, no commercial investor will consider such investments. The following analysis is therefore presented to clarify the impact of reinvestment on IRR and NPV and to resolve those controversial issues.

a. First, the IRRs and NPVs estimated by CAS/DCF method are discussed (Table 4):

- For projects A and B, there are two IRRs of 6.6% and 36.55% (9.9% and 32.2%)[8]. The NPVs also are zero twice, once at 6.6% and again at 36.55% (9.9% and 32.2%).
- Assuming a hurdle rate of 10%, the NPV is $27.80 million ($0.4million). NPV at 20% is $52.47 million ($15.6 million) which is the highest NPV. Such a strange value of NPV (increases with higher discount rates) is typical of non-monotonic NPV function, normally associated with NNCF (see Arjunan 2019; Merlo 2012). The usual discounting using the hurdle rate (a point estimate) fails to capture such highest but spurious NPV dynamics and would not have even noticed the zero NPV at 6.6% (9.9%). When the NPV is zero at discount rates 6.6% (9.9%), it is abnormal to get a positive NPV of $52.47 million ($15.6 million) at 20%. Both NPV and IRR estimates are spurious and not helpful in decision to invest or not. It is not rational to argue that IRR suffers with multiple rates (two here) without acknowledging the fact that NPV does suffer with multiplicity of NPVs (several variants) and inconsistent NPVs caused by the NNCF.

b. Second, the IRRs and NPVs estimated by MCAS method are discussed (Table 4):

A MCAS method (discussed under methodology) eliminates reinvestment income and resolve the problem of multiple IRR and leads to a unique IRR. The results derived by the MCAS method is discussed here:

- With elimination of the controversial reinvestment income associated with NNCFs projects A and B, there are unique IRR of -9.0% and -12.1%, respectively. The negative IRR realistically

[8] Figures in parenthesis correspond to project B.

reveals approximate capital loss of 9% or 12% (of the capital invested) and therefore these investments must be rejected. The NPVs at 10% are also negative in both projects A and B, under MCAS method.

- With the current practice of DCF or CAS methods, had the “NPV rule” or ÏRR rule” adopted, both A and B would have been accepted as the NPVs at 10% are positive and at least one of the IRRs are above hurdle rate. The result indicates capital loss and therefore the NPV rule is misleading and IRR confusing.

In summary, both IRR and NPV suffers with multiple estimates with NNCF data but then the criticism is equally, rather more, applicable to NPV. As pointed earlier, the DCF or CAS analysis using unorthodox or abnormal or non-normal NCF, with negative or zero or negligible undiscounted value, affects the credibility or rationality of the analysis. The limitation of the input data (NNCF) and the DCF/CAS method must be clearly understood. The MCAS method, for NNCF as well as NCF, offers consistent and realistic estimates and resolves the problem of multiple IRR.

4.3 Mutually Exclusive Projects

Investment decision, to accept or reject or rank mutually exclusive projects, based on contradicting NPV or IRR criteria is another controversial issue. The proponents of NPV rule argue that the IRR is not suitable for evaluating mutually exclusive projects (for e.g. Osborne 2010, Kierulff 2012, World Bank 2013, ADB 2017). The validity of this argument is critically evaluated in this section.

In addition to the estimation of IRR and NPV and profiles of NPV and BCR are prepared for all projects at various discount rates including an assumed hurdle rate of 10% and at IRRs of both projects (the IRR of project A is used as discount rate for project B and vice versa). The results are furnished in Table 5 in five parts: 1. NCFs for three sets of mutually exclusive projects; 2. Estimated IRR, NPV, Cross-over rate and BCR (PI); 3. NPV Profiles of mutually exclusive projects; 4. BCR Profiles of mutually exclusive projects; and 5. ROC and ROIC Profiles of mutually exclusive projects. The results are summarized and discussed here.

a. Among the three sets of mutually exclusive projects, in the first set the IRR, NPV and BCR consistently support project B. In the case of second set, IRR supports project 'B' but NPV supports 'A'. Similarly, under case 3, IRR supports project 'A' whereas NPV supports project 'B'. The NPV rule prefer the NPV as the best criteria. Accordingly, project 'A' must be accepted under second set and project 'B' under the third set; but their NPVs are zero or negative and BCRs '1.0' or below '1.0' at the highest IRR (as discount rate or with higher CoC) achieved by their counterpart projects. From that context, projects 'B, B and A' based on IRR can also be considered or must be evaluated. This analysis is further expanded before acceptance or rejection of NPV rule for mutually exclusive projects.
b. A review of the mathematical relationships between NPV, BCR and IRR are furnished here below.

R < IRR	NPV = POSITIVE	BCR = >1.0	PV of Benefits exceeds PV of Costs – Profitable
R = IRR	NPV = 0	BCR = 1.0	PV of benefits = PV of costs - Break-even
R > IRR	NPV = NEGATIVE	BCR = <1.0	PV of costs exceeds PV of benefits – not profitable

Where R is the discount rate which is either CoC or WACC or risk adjusted WACC.

According to the mathematical relationship, NPV is zero at IRR and BCR is 1.0 at IRR as discount rate. NPV continues to remain positive for all discount rates below IRR and but shift to negative territory for discount rates above IRR. This indicates that, at IRR, the PV of benefits offsets the PV of capital invested. The positive NPV at hurdle rate indicates that the NCF stream is not yet fully utilized and has the potential to generate rate of return more than the hurdle rate. The NPV, therefore, reveals the cut off point for acceptance and not indicates the maximum ROIC achievable with the NCF generated by the investment.

c. At IRR, the capital invested is fully recovered (ROC) with a return equivalent to IRR (ROIC) and therefore, the NPV is zero (the unutilized NCF). Both NPV and BCR are point estimates (at hurdle rate) and they change with changes in discount rates. With increasing WACC or CoC or discount rates, NPV and BCR will likely to change from profitable to not profitable.

d. A review of the NPV and BCR profiles for each of the three sets of projects reveals a different picture. Based on the results presented in Table 5, had the point estimate (at cost of capital) NPV is used, projects B, A and B under the first, second and third sets of projects, respectively, would have been accepted. However, a review of the NPV and BCR profiles reveal:

 - Under set I, project B is supported by IRR, NPV and BCR with highest values under various discount rates. The project 'B' with highest IRR is therefore selected.
 - Under set II, IRR supports project 'B'. The NPV and BCR profiles indicates that project 'A' has higher NPV and BCR at discount rates below 15% and project 'B' has higher NPV and BCR for discount rates above 15%. In other words, when the CoC is less than 15%, project 'A' will be preferable but then with CoC more than 15%, project 'B' must be selected. A rational investor will prefer project 'B' with higher IRR because with increasing discount rate up to IRR level, the NPV and BCR will be also higher with project

‘B’ and not with the project ‘A’. The IRR reveals the potential profitability of the investments and therefore a better criterion than NPV.

- Under set III, based on NPV rule, project ‘B’ must be selected. The profiles of NPV and BCR indicate that NPV and BCR of project ‘B’ is higher when the discount rate (CoC) is below 15% but with discount rates (CoC) above 15%, the NPV and BCR of project ‘A’ is higher than project ‘B’. Here again, the NPV rule is invalid and the ranking goes with IRR.

e. With increasing discount rates or CoC, projects with higher NPV and lower IRR tend to have low NPVs and BCRs; whereas projects with higher IRRs and low NPVs tend to have higher NPV and BCR with higher CoC or discount rates. The timing of cash flow might have naturally caused such an outcome. But then, the time value concept and discounted cash flow analysis is fundamentally based on timing of cashflow and therefore the results are supported by theory too.

f. The main purpose of capital budgeting is to evaluate whether the cashflow of an investment is adequate to recover the capital invested (return of capital – ROC) and to recover a desired or maximum return on the invested capital (return on invested capital – ROIC). The NCF must be fully utilized to pay-off ROC and to maximize the ROIC. That’s what the function of capital investment to maximize the ROIC and to recover the ROC. A ROC and ROIC analysis is undertaken and the results are available in Table 5 and a summary of the findings discussed:

- As per the ROIC analysis, using the criteria of ROC and ROIC, project B, B and A are accepted. The return on invested capital (ROIC) is one measure of a company’s capital efficiency and that the ROIC analysis can provide insight into the sources of a company’s competitive advantage (see Mauboussin and Callahan 2014; Damodaran, 2007). Damodaran (ibid) also argued that growth unaccompanied by excess returns (ROIC) creates no value. The

ROIC criterion is therefore important both for selection of investment and for ranking of mutually exclusive investment

Table: 5. Estimated IRR, NPV, cross-over rate and profitability index (BCR)

Years / Criteria	**Silber (2016) – Set I**		**Osborne (2010) – Set II**		**Damodaran (2010) – Set III**	
	Project A	Project B	Project A	Project B	Project A	Project B
1. NCFs for the three sets of mutually exclusive projects						
0	-1000	-1000	-100	-100	-1000	-10000
1	0	320	30	50	350	3000
2	0	320	35	50	450	3500
3	300	320	45	40	600	4500
4	600	320	60	20	750	5500
5	900	320	-	-	-	-
2. Estimated IRR, NPV, Cross-over rate and Profitability Index						
IRR	14.7%	18.0%	22.0%	25.4%	33.7%	20.9%
NPV $ at 10%	194.03	213.05	30.99	30.49	653.13	2757.33
Cross-over rate		8.8%		10.7%		19.3%
Profitability Index (BCR)	1.19	1.21	1.31	1.30	1.65	1.28

3. NPV Profiles of mutually exclusive projects selected								
Silber (2016)			Osborne (2010)			Damodaran (2010)		
Discount Rate	Project A: NPV	Project B: NPV	Discount Rate	Project A: NPV	Project B: NPV	Discount Rate	Project A: NPV	Project B: NPV
10.0%	194.03	213.05	10.0%	30.99	30.49	10.0%	653.13	2757.33
14.7%	**0.00**	**81.20**	20.0%	4.28	9.18	15.0%	467.94	1358.66
15.0%	-12.23	72.69	**22.0%**	**0.00**	**5.66**	20.0%	313.08	187.11
18.0%	**-115.50**	**0.00**	25.0%	-5.98	0.67	**20.9%**	**288.37**	**0.00**
20.0%	-175.35	-43.00	**25.4%**	**-6.78**	**0.00**	30.0%	71.20	-1647.35
			30.0%	-14.72	-6.74	**33.7%**	**0.00**	**-2188.64**
4. BCR Profiles of mutually exclusive projects selected								
Discount Rate	Project A: BCR	Project B: BCR	Discount Rate	Project A: BCR	Project B: BCR	Discount Rate	Project A: BCR	Project B: BCR
10.0%	1.19	1.21	10.0%	1.31	1.30	10.0%	1.65	1.28
14.7%	**1.00**	1.08	20.0%	1.04	1.09	15.0%	1.47	1.14
15.0%	0.99	1.07	22.0%	**1.00**	1.06	20.0%	1.31	1.02
18.0%	0.88	**1.00**	25.0%	0.94	1.01	20.9%	1.29	**1.00**
20.0%	0.82	0.96	25.4%	0.93	**1.00**	30.0%	1.07	0.84
			30.0%	0.85	0.93	33.7%	**1.00**	0.78
5. Return of Capital (ROC) and Return on Capital (ROIC) of investments								
ROC	1000	1000		1000	100		1000	10000
ROIC/IRR	14.7%	18.0%		22%	25.4%		33.7%	20.9%
Rank	2	1		2	1		1	2
Decision	Reject	Accept		Reject	Accept		Accept	Reject

- Investments B, B and A have fully recovered the capital invested (ROC) and achieved the highest ROIC (IRR is the highest) among the mutually exclusive projects in each set. Projects A, A and B are rejected as their ROIC (IRR) are lower than their counterparts. As discussed earlier, NPV, being the PV of the unutilized balance of the NCF, fails to provide a better information than IRR. Cross over rate, as the discount rate, makes the NPV same for both mutually exclusive projects. PI (equal to BCR) is another output from the DCF using the same NCF data. These two measurements could not have solved the problems.

Based on this analysis, it is concluded that the NPV rule must be revisited and the IRR is a better criterion than NPV to rank mutually exclusive and independent investments.

4.4 IRR as return on total capital vs return on declined or balance of capital

IRR by the DCF method indicates the return on declining capital or balance of capital (see Kelleher and MacCormack, 2004). The CAS method enables the analysts to estimate IRR on the declining capital or on total capital, if they decide to do so. The IRR on the declining capital and on total capital invested for two hypothetical NCFs are estimated using CAS method and furnished in Table 6.

As could be seen (table 6) the IRRs for NCF 1 are 14.7% and 11.3% on declining or balance of capital (normal IRR) and the total capital invested, respectively. For NCF 2, the estimated IRRs are 24.9% and 20% on declining capital and total capital, respectively. With return on total capital, the interest income or return is constant for all years. The investment executives may be more convinced with return on total capital invested. There are two important points that must be noted. First, the declining capital is due to the recovery of part of the capital every year. The recovered capital can be invested elsewhere that would generate return (opportunity cost of capital). That income will compensate for the difference between return on declining balance and on the total capital. Second, in both cases of return on total or balance of capital, the total inflows are $145000 and $18,000 under the cases 1 and 2, respectively. The return remains constant and is only reallocated to various years and the timing of return is altered.

Table: 6. IRR as Return on Declining Balance of Capital vs Total Invested Capital

NCF 1: CAS A	-100000	25000	30000	40000	50000.0
Opening Capital		-100000	-89697.6	-72881.1	-43592.9
Interest at 14.7%		-14697.6	-13183.4	-10711.8	-6407.1
Income		25000.0	30000.0	40000.0	50000.0
Closing Balance		-89697.6	-72881.1	-43592.9	0.0
IRR on Declining Capital		**14.7%**	**Total Income flow = 145,000**		
NCF 1: CAS B	-100000	25000	30000	40000	50000.0
Opening Capital		-100000	-86250.0	-67500.0	-38750.0
Interest at 11.3%		-11250.0	-11250.0	-11250.0	-11250.0
Income		25000.0	30000.0	40000.0	50000.0
Closing Balance		-86250.0	-67500.0	-38750.0	0.0
IRR on Total Capital		**11.3%**	**Total Income flow = 145,000**		
NCF 2: CAS A	-10000	3000	4000	5000	6000.0
Opening Capital		-10000	-9488.8	-7850.4	-4804.3
Interest at 24.9%		-2488.8	-2361.6	-1953.8	-1195.7
Income		3000.0	4000.0	5000.0	6000.0
Closing Balance		-9488.8	-7850.4	-4804.3	0.0
IRR on Total Capital		**24.9%**	**Total Income flow = 18,000**		
NCF 2: CAS B	-10000	3000	4000	5000	6000
Opening Capital		-10000	-9000.0	-7000.0	-4000.0
Interest at 20%		-2000.0	-2000.0	-2000.0	-2000.0
Income		3000.0	4000.0	5000.0	6000.0
Closing Balance		-9000.0	-7000.0	-4000.0	0.0
IRR on Declining Capital		**14.7%**	**Total Income flow = 18,000**		

4. Summary and Conclusion:

In this paper, the validity of the NPV rule and common criticism against IRR is evaluated. The assumption of reinvestment of intermediate income, multiple IRR (including no or zero or negative IRR), the right criterion to rank mutually exclusive investments and the superiority of NPV vs IRR as a criterion, are evaluated and presented. This analysis makes use of data from different sources including some hypothetical cases. The approach primarily involves estimation of NPV, BCR and IRR by DCF as well as CAS and MCAS methods under different scenarios and discount rates. The estimated indicators are compared and investigated. The main findings are summarized here below:

a. **No or Zero or Negative IRR:** In investment analysis, if the NCF does not have a single negative flow (in year 0 or 1), then there will be no IRR because there is no ROC needed. In such cases the NPV collapse into PV of benefits. Mathematically this is consistent with the Descartes' rule of sign[9] as there is no sign changes in this case (all positive cash flow, no negative flow). Similarly, if the sum of un-discounted NCF is '0' or negative (no profit or net loss), then there is no IRR or negative IRR which truly reflects the intrinsic value of the NCF. In such cases, 'no' IRR does not mean IRR is an unsuitable criterion. NPV can be used to estimate the present value only for valuation and not as an investment criterion.

b. **Reinvestment of intermediate income**: With normal NCF and some of the NNCF data, neither IRR nor the NPV involves reinvestment at rates equivalent to IRR or at hurdle rates, respectively. In those case, the present analytical evidence rejects the assumption of reinvestment at IRR or at hurdle rate (CoC). There are some normal NCF cases where the reinvestment affects the NPV and makes it spurious but not the IRR. Under the CAS method, positive OB for one or more years is possible with some NNCF (not all NCCF). Those positive OB triggers the reinvestment (interest income on positive OB) that leads to multiple IRR and spurious NPV too.

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[9] the maximum number of positive real number solutions (roots) of a polynomial equation in one variable based on the number of times that the signs of its real number coefficients change.

c. **Multiple IRR and Spurious NPV**: With NNCF data both IRR and NPV suffer with multiple IRR and spurious NPV. DCF analysis using unorthodox or NNCF affects the consistency of both IRR and NPV. Reinvestment of intermediate income and the associated multiple IRRs and spurious NPVs are symptoms of some of the NNCF data and not associated with all NNCF and normal NCF. The NPV will be spurious and will be zero as many times as there are IRRs. The limitation of the input data (NCF) and the DCF method are the causes and the effects are inconsistent IRR and NPV. This must be clearly acknowledged rather than arguing as to which estimate (IRR and NPV) is appropriate. The MCAS method eliminates the reinvestment income and thereby resolves the problem of spurious NPV and multiple IRR leading to a unique IRR. The PI, which is BCR, being another estimate from the DCF method cannot solve the problem.

d. **NPV vs IRR:** Between NPV and IRR, IRR is the only indicator that is estimated by fully utilizing the NCF stream. IRR explicitly indicates whether the NCF is fully utilized to recover the ROC and ROIC. NPV makes use of a part of NCF to cover the capital and the indicate the balance of the NCF. When NPV is positive or zero or negative that indicates that the NCF is not fully unutilized or fully utilized or not adequate to cover the cost of capital, respectively. The higher the NPV the more is the unutilized NCF that is still available to achieve a higher ROIC (IRR).

e. **Mutually Exclusive Investments:** Among the mutually exclusive projects, NPV rule prefers investments with highest NPV at hurdle rate. NPV rule ignores and highest IRR or ROIC for the given NCFs whereas the IRR consistently reveals the projects with highest ROIC. When the project with higher NPV at hurdle rate is discounted by the rate, equivalent to the IRR achieved by the counterpart project, the NPV becomes negative. Whereas the higher IRR project's NPV remains zero. These findings indicate that IRR is the preferred criterion for mutually exclusive projects. The analysis is extended to review the NPV and BCR profiles and the ROC and ROIC profiles.

The review of NPV and BCR profile at various discount rates indicates that investment with highest NPV at hurdle rate (10%), could only achieve a low NPV (or zero or negative NPV) and a low BCR (or 1.0 or < 1.0), compared with their counterparts, with increasing discount rates above hurdle rate. Whereas investments with higher IRR achieve higher NPV and BCR than their counterparts with higher NPV and BCR at hurdle rates. The profile review of NPV and BCR supports IRR as the best criterion.

Next a review of ROC and ROIC is undertaken. The corporate management always targets to recover the capital invested (ROC) and to maximize the return on investment (ROIC) or the profitability of the investment. They consider the NPV at hurdle rate (10%) as the bottom line. The review indicates that projects with highest IRR fully recovers the invested capital(ROC) and generated the highest return on investment (ROIC = IRR) The present findings support the ROC and ROIC combined criterion to rank or select among the mutually exclusive projects and for independent projects too.

These findings reject the conventional wisdom of acceptance of projects based on NPV criteria for the mutually exclusive projects. IRR explores and estimates the potential maximum return even beyond the hurdle rate. IRR is, therefore, the best indicator as that reveals the exact rate of return which is above the hurdle rate.

f. **IRR on total vs balance of capital:** The CAS method enables to estimate the IRR as return on total capital as well as on the balance of capital. The IRR on balance of capital is higher than the IRR on the total capital. However, the balance of capital reveals that the recovered capital each year is still available to invest elsewhere and to get return on other investments (opportunity cost of capital). The income flow remains the same but the recovery of ROC and ROIC are reallocated and therefore the analyst can choose which ever return they prefer or to use both.

In conclusion, these findings make out a case to consider revisiting the NPV rule and to recommend IRR as a best criterion for both independent and mutually exclusive capital investment projects. NPV is important wherever the net present value is needed.

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